Quasi-Stable Dynamics of Complex Systems: A PhD Project

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Abstract

We combine geometric data analysis and stochastic modeling to describe the collective dynamics of complex systems and focus on financial markets. We identify the dominating variable and extract its explicit stochastic model. We analyze dynamically distinct market states and quantify system behavior within the states.

This work is a part of Yuriy Stepanov's PhD thesis at University Dusiburg-Essen under the supervision of Prof. Thomas Guhr and PD Rudi Schäfer in collaboration with the group of J. Peinke. It is financially supported by the Hans-Böckler-Foundation and published in Ref. [1] with a related work published in Ref. [2].

Geometric Approach and Market States

- Analyzed data: rolling correlation matrices $C(t)$ of the $K = 307$ companies in the S&P500® Index. $C(t)$ has only $d = (K^2 - K)/2$ independent entries.
- Principal component analysis applied to $C(t)$ seen as correlation vectors $\vec{c}(t) \in \mathbb{R}^d$.
- First principal component

$$\vec{v}^{(1)} \approx (1, 1, ..., 1)/\sqrt{d}. \qquad (1)$$

- The projection on $\vec{v}^{(1)}$

$$\langle \vec{c}(t), \vec{v}^{(1)} \rangle = \frac{1}{\sqrt{d}} \sum_{i=1}^{d} c_i(t) = c(t)\sqrt{d}, \qquad (2)$$

is the mean correlation coefficient $c(t)$ times $\sqrt{d}$.

- Market states are obtained by clustering $C(t)$ following Ref. [3].

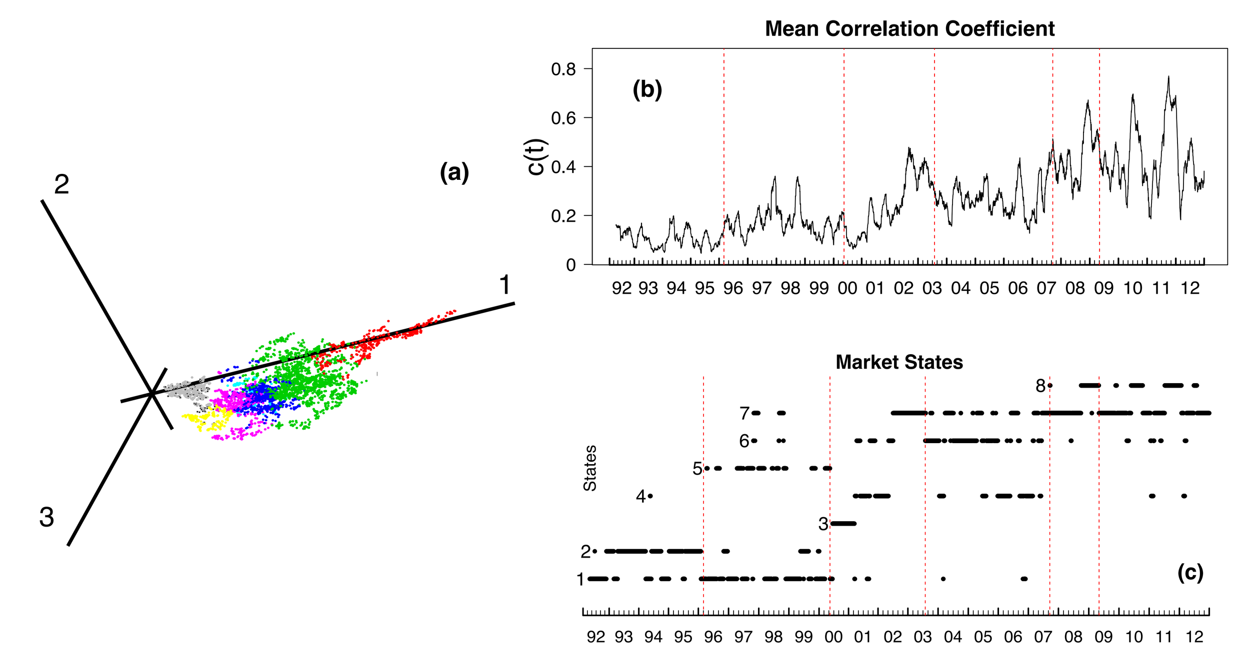


Figure 1: (a) Projection of $\vec{c}(t)$ onto the first three principal components, (b) the time series of $c(t)$ and (c) time evolution of the market states.

Stochastic Modeling

- Mean correlation $c(t)$ is modeled by an Ito-Process

$$\dot{c}(t) = f(c, t) + \sqrt{g(c, t)}\,\Gamma(t). \qquad (3)$$

- Explicit stochastic model is extracted from the data following Refs. [4-6].
- Instead of the drift f, potential functions

$$V(c, t) = -\int^{c} f(x, t)\,\mathrm{d}x \qquad (4)$$

are considered. Maxima and minima of V are system fixed points.

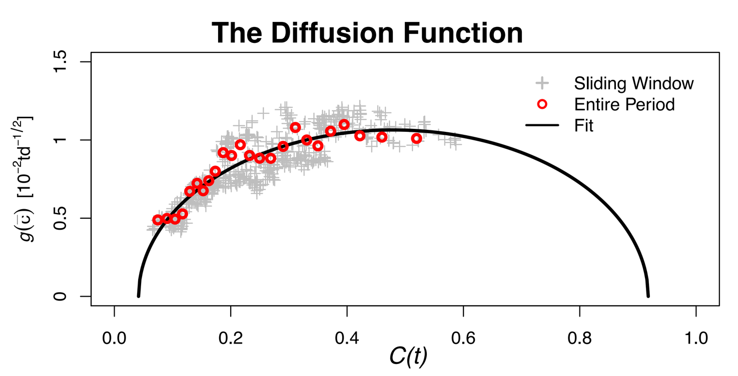
Diffusion

- Empirical data fitted by the time-independent function

$$g(c) = \lambda\sqrt{(c - c_{\min})(c_{\max} - c)}, \qquad (5)$$

with $\lambda = 0.0245\ \mathrm{td}^{-1/2}$, $c_{\min} = 0.042$, $c_{\max} = 0.918$ yield characteristic time scale

$$t_0 = \lambda^{-2} = 1666 \text{ trading days}. \qquad (6)$$



The characteristic time scale t_0 agrees with the slow correlation dynamics time scale found in Refs. [7-8].

Figure 2: Diffusion function $g(c, t) \equiv g(c)$.

Drift Potential Functions

- Drift and thus potential function is strongly time dependent and sensitive to economically distinct events.

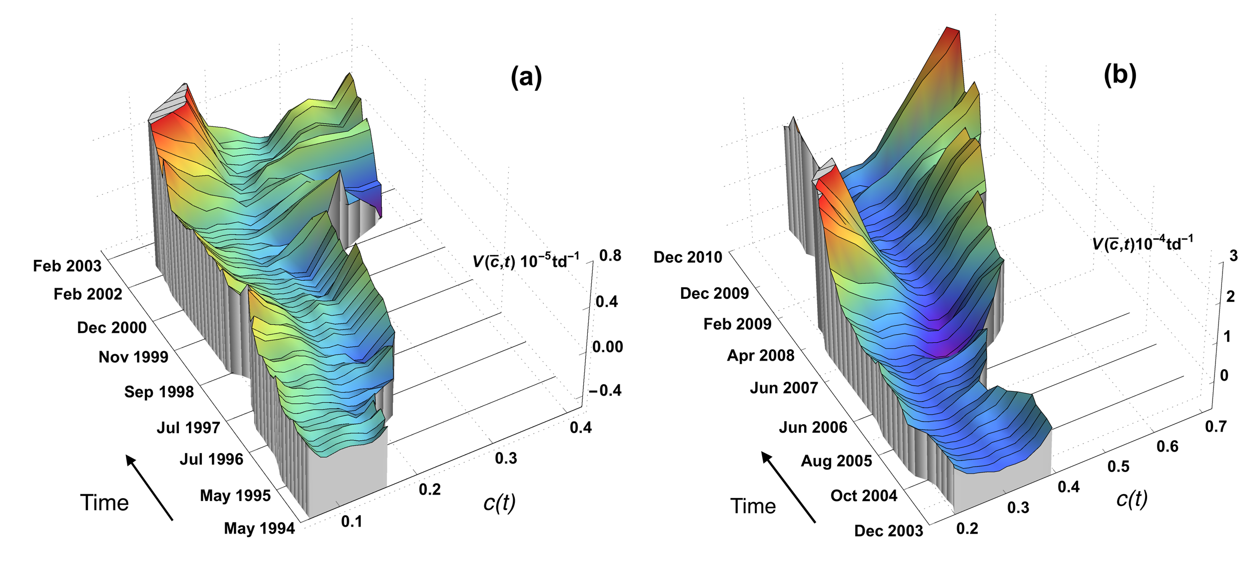


Figure 3: Time evolution of the drift potential function $V(c, t)$.

Market States Dynamics

- Market states dynamics is obtained by restricting drift estimation.
- The absolute increments

$$\Delta(t) = |c(t + 1) - c(t)|, \qquad (7)$$

show that changes of $c(t)$ are on average larger while state transitions.

- Potential functions classify market states according to the stability.
- The market is 'hopping' from state to state in the potentiallandscape.

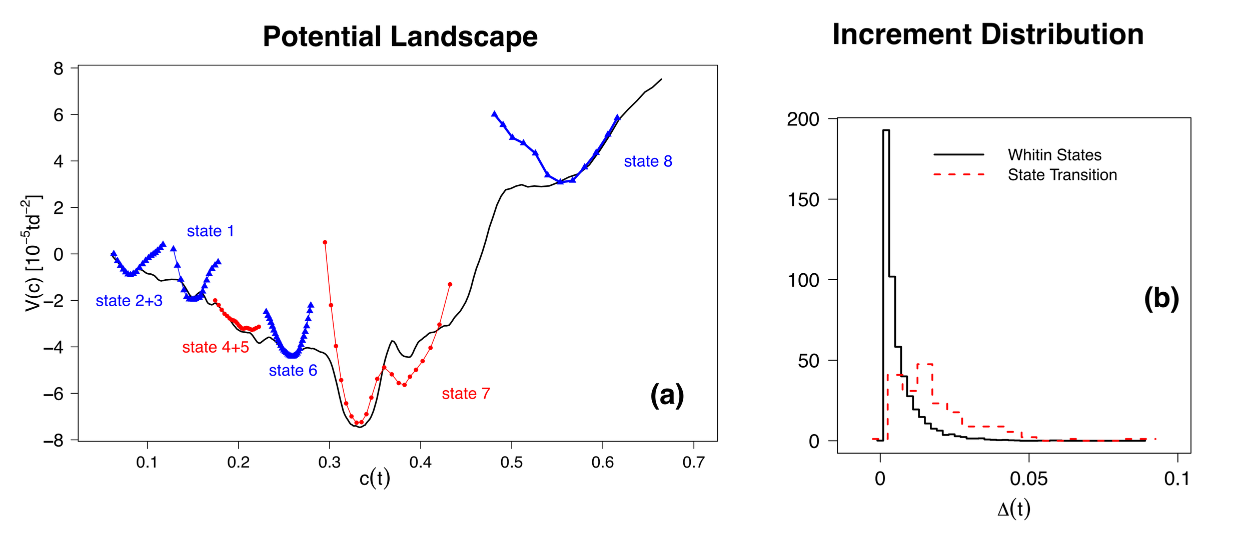


Figure 4: (a) Potential landscape, (b) the increment distribution according to Eq. (7).

Conclusion and Outlook

- Geometric data analysis and stochastic modeling give access to low dimensional description of complex systems.
- For financial markets, large stock portfolio correlation dynamics is given by the mean correlation coefficient $c(t)$.
- The non-stationary market dynamics is due to to changes in the deterministic term f in Eq. (3).
- Results may be used to detect collective trends on the market and for trading index derivatives.

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Multi-Asset Correlation Dynamics with Application to Trading: An Internship Project

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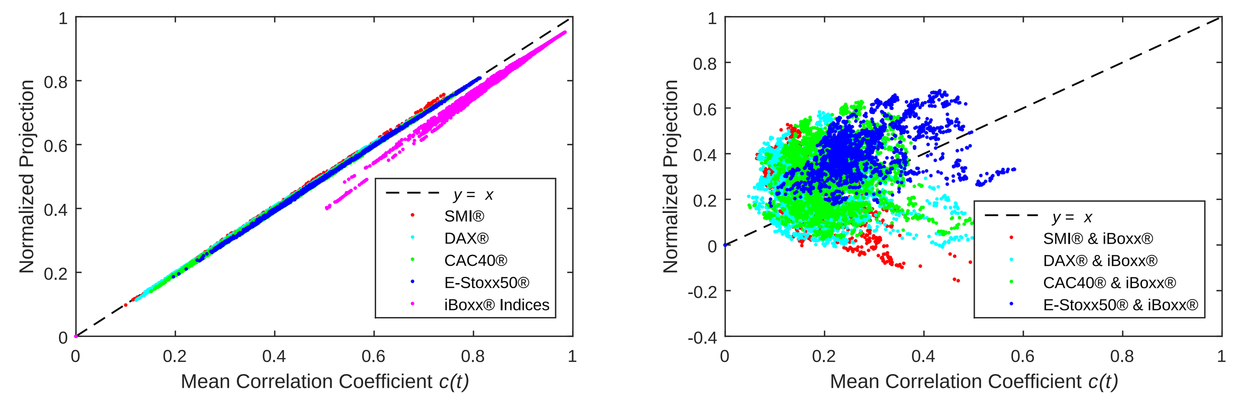
Multi-Asset Portfolio Correlation Dynamics

Abstract - We study correlation dynamics of European equities and iBoxx® [9] bond indices applying techniques of Refs. [1,3]. We show that the mean correlation coefficient is the dominating one-dimensional variable for all equity portfolios, which is not the case for multi-asset portfolios. As a direct application, we use correlation extreme values for trading equity index futures.

This analysis is a summary of Yuriy Stepanov's internship project at Quant.Capital GmbH under the supervision of Tarek Abou-Zeid and Erik Wellner.

Data Analysis

- Geometric analysis of Ref. [1] applied to stock compoments of four European equity indices DAX®, E-Stoxx50®, SMI®, CAC40® and a portfolio of 12 iBoxx® indices.
- The projection onto the first principal component is given by the mean correlation $c(t)$ for equities and with small error deviations for the iBoxx® portfolio.
- This is not the case for a multi-asset portfolio of equity index stock members and iBoxx® bond indices.
- Equations (1) and (2) represent a generic feature for equities.



(a) (b)

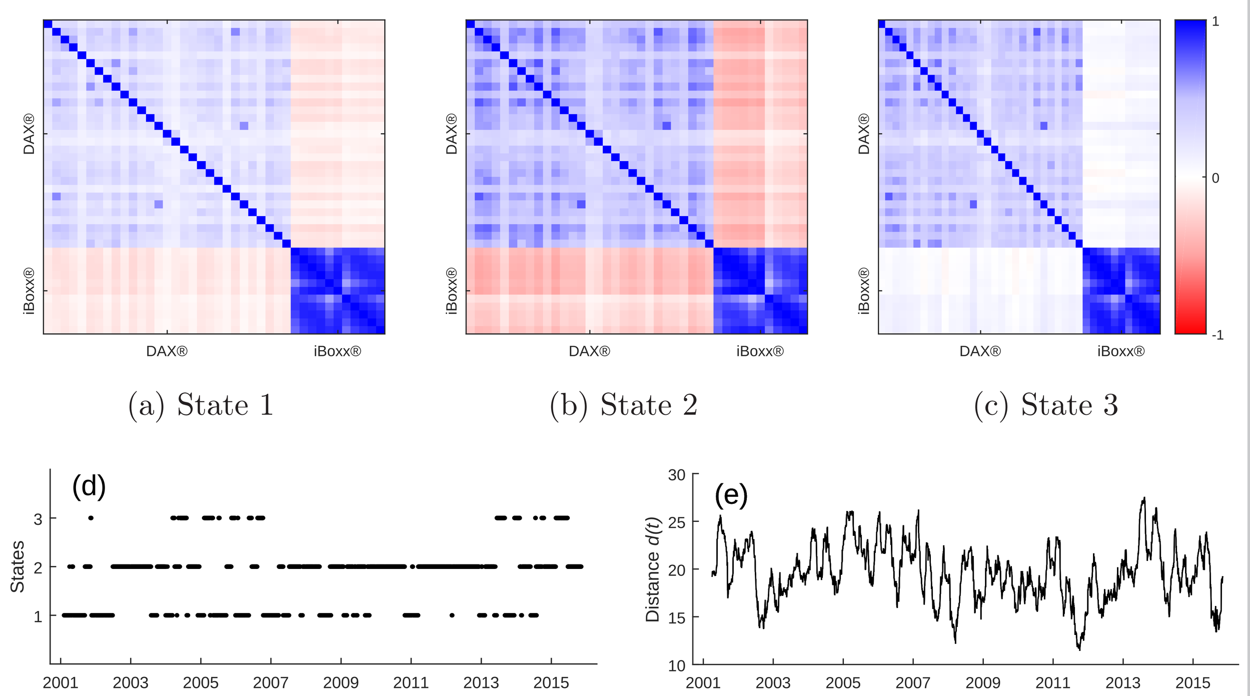
Figure 5: Graphical validation of Eq. (2): (a) for single-asset portfolios and (b) for multi-asset portfolios.

Correlation Dynamics

- There are 6 multi-asset market states, obtained by clustering the correlation matrices of DAX® stocks combined with the 12 iBoxx® indices, resulting from 3 main groups.
- Cluster centers have a strong block structure. The asset classes are either anti-correlated or uncorrelated.
- Market dynamics is quantified by the distance

$$d(t) = \|C_0 - C(t)\|, \qquad (8)$$

of $C(t)$ to the reference correlation matrix C_0, with $+1$ entries within the DAX® and iBoxx® blocks and -1 in the crossing blocks.



(a) State 1 (b) State 2 (c) State 3

Figure 6: (a)-(c) The average multi-asset correlation matrices of the three leading clusters. (d) Multi-asset market states dynamics and (e) $d(t)$ according to Eq. (8).

Trading Strategy using Correlation Signals

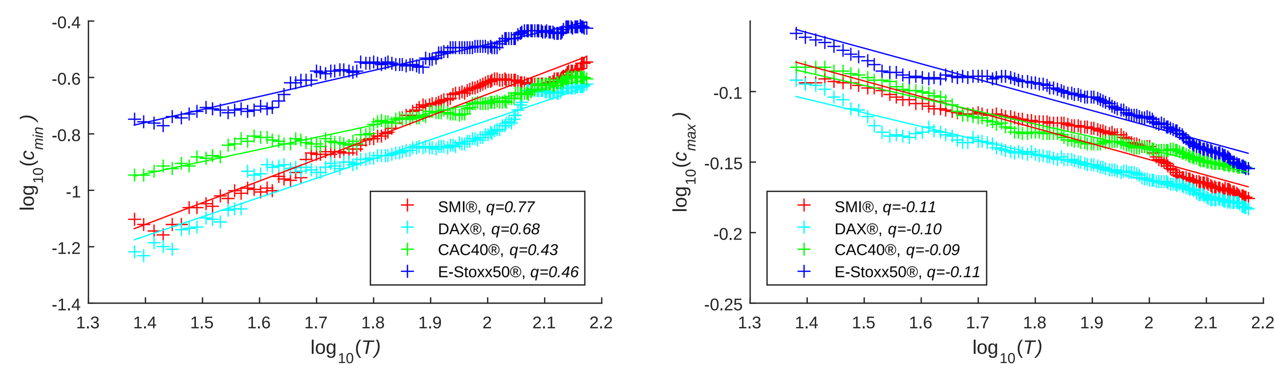
Correlation Extreme Values:

- For equities Eq. (2) implies that extreme values of the mean correlation signalize collective behavior changes and thus trend changes of the corresponding index.
- Mean correlation extreme values are fitted by a power law

$$c_i(T) = c_0^i T^q, \qquad (9)$$

with $i = \min, \max$ for a given look back T.

- The power law exponent q is remarkably constant for the high mean correlation values.



(a) Minimal mean correlation (b) Maximal mean correlation

Figure 7: Extreme values of the mean correlation coefficient and fits according to Eq. (9).

Index Future Trading Strategy:

- A trading position is opened or closed if $c(t)$ crosses a certain high or low threshold value $\tilde{c}$.
- The optimal threshold value may differ for long and short positions.
- Equities generic features suggest that such strategy is robust across different indices.

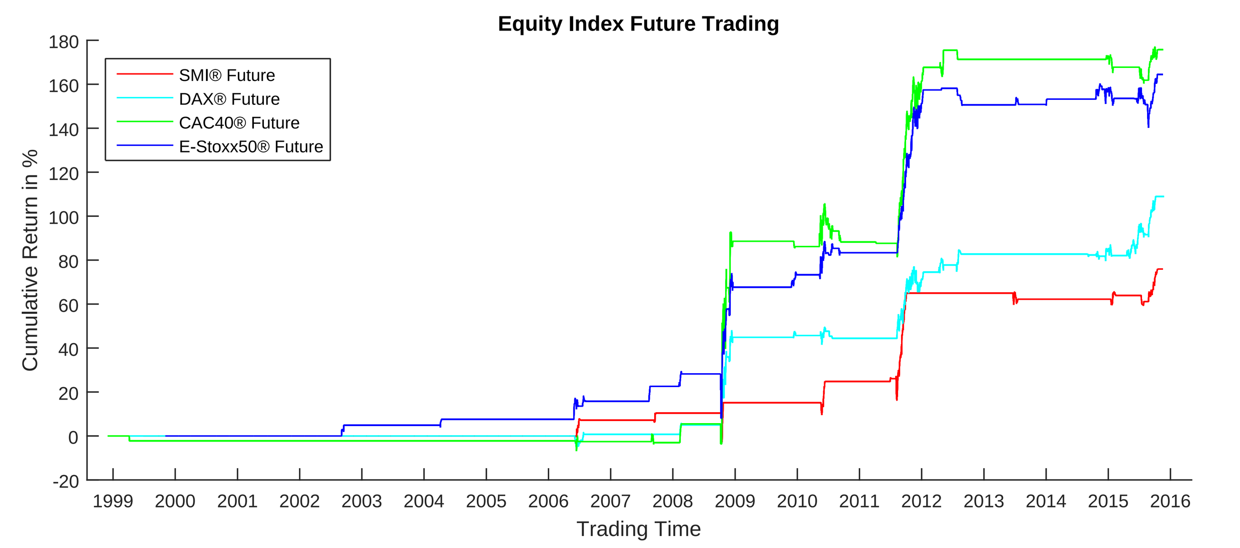


Figure 8: Index future trading cumulative return.

Strategy Parameters Optimization:

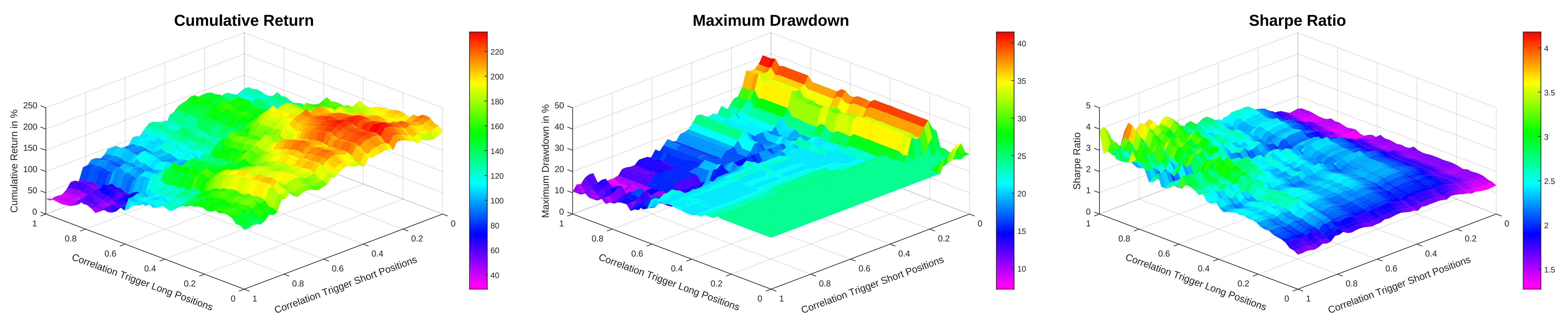


Figure 9: Strategy optimization surface plots for the E-Stoxx50® Index. Correlation trigger values 1 correspond to high threshold values and 0 correspond to low threshold values.