Analysis of leptokurtosis in model distributions and simulated noises

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ABSTRACT

We modified threshold Sieczka-Hołyst model to cover all known up to now stylized facts. This modifications are as follows: (i) the impact of non-Gaussian noise (hierarchical Weierstrass-Mandelbrot random walk) on decision of each agent, (ii) if saturation of magnetization is reached then reset of the system is made.

Introduction

Modelling of financial markets is a burning issue, especially recently, because of still continuing Global Financial Crisis. In this work we considered a threshold model, proposed by Paweł Sieczka and Janusz Hołyst [2, 3] as a very promising model of financial markets. Our work was based on stochastic dynamics method computer simulations.

Goal

Main goal of our work was reconstruction of Sieczka and Hołyst results, and then systematic investigation of model properties. We covered also one of possible modification of the model, and showed the results it produces.

Assumptions of the model

In the threshold model we consider N interacting agents. Each of them can take one of three actions: buy, sell or wait (stay inactive), represented by value of three-state spin variable s_i, $i = 1, ..., N$. These values are respectively: +1, -1 and 0. The agents interact according to interaction matrix J_{ij}. Each agent is influenced only by its four nearest neighbors, with equal strength $J > 0$.

Each agent makes investment decision according to equation:

$$s_i(t) = sign_{\lambda|M(t-1)|}\left(\sum_{j=1}^{N} J_{ij}\cdot s_j(t-1) + \sigma\eta_i(t)\right),$$

where $sign_q(x)$ equals 1, 0 or -1 respectively for $x > q$, $-q < x < q$, or $x < -q$, and $\eta_i(t)$ is a random variable from the standard normal. Magnetization is defined as:

$$M(t) = \frac{1}{N}\sum_{i=1}^{N} s_i(t).$$

Asset's price and rate of return

Asset's price:

$$P(t) = P_0(t)e^{M(t)}.$$

An assumption made by Authors was:

$$P_0(t) = P_0 = const\ ,$$

so we obtain a logarythmic rate of return:

$$r(t) = \ln\left(\frac{P(t)}{P(t-1)}\right) = M(t) - M(t-1).$$

Generalization – rate of return with time lag τ:

$$r_\tau(t) = M(t) - M(t-\tau).$$

Simulation's results*

* Figs 1 – 4 are our reproduction of Sieczka and Hołyst's figures in [2, 3]

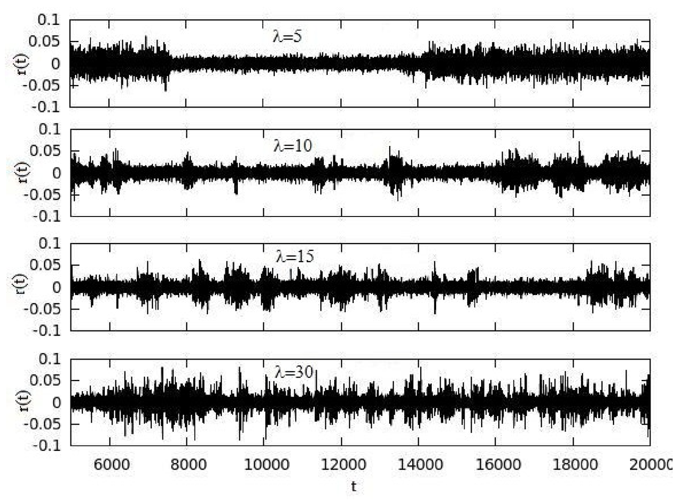

Fig. 1. Returns $r(t)$ in time t ($J = 1$, $\sigma = 1$, different λ). Volatility clustering.

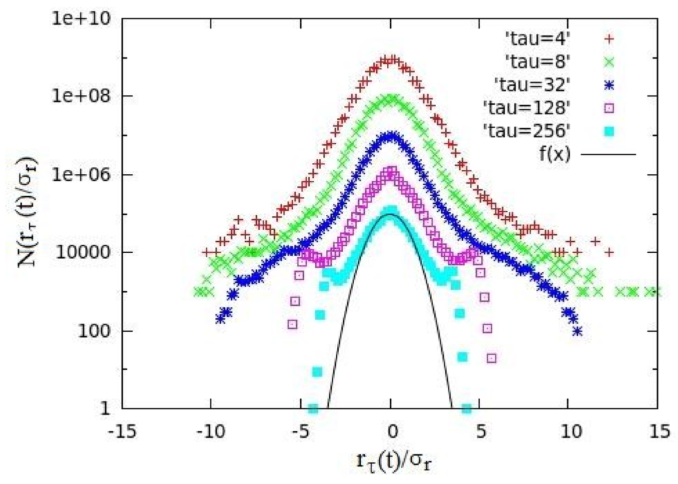

Fig. 2. Histograms of returns $r_\tau(t)$ ($J = 1$, $\sigma = 1$, $\lambda = 10$, different τ). Fat tails are slimming down.

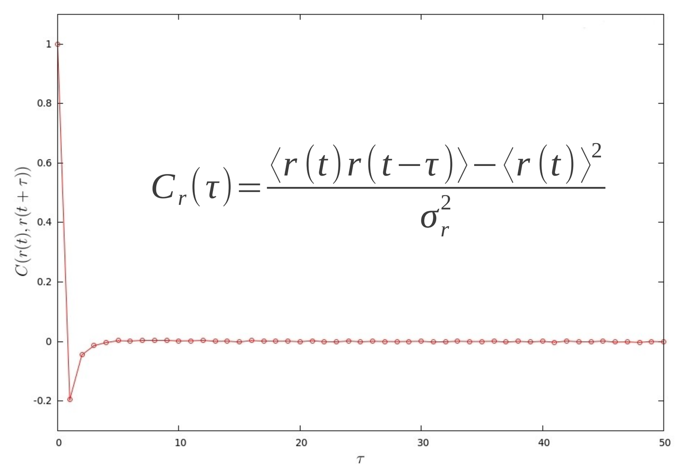

Fig. 3. Autocorrelation function of return r(t) ($J = 1$, $\sigma = 1$, $\lambda = 15$).

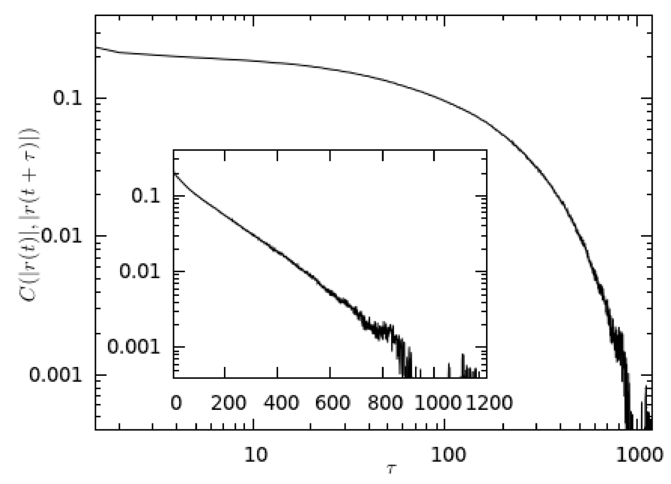

Fig. 4. Autocorrelation function of absolute return |r(t)| is exponential ($J = 1$, $\sigma = 1$, $\lambda = 15$).

Modification of threshold model

We proposed the modification of the model:
(i) non-Gaussian noise $\sigma\eta_i(t) \to b_0 b^j(t)$ (hierarchical Weierstrass-Mandelbrot random walk),
(ii) if saturation of magnetization is reached then reset of the system is made.

Hierarchical Weierstrass-Mandelbrot random walk:

$$p(x) = \left(1 - \frac{1}{N}\right)\sum_{j=0}^{\infty}\frac{1}{N^j}\cdot\frac{1}{2}\delta(|x| - b_0 b^j),$$

$$b_0 > 0, N > 1, b > 1.$$

We took: $b_0 = 0.05$, $N = 3$, $b = 2.9$ and $J = 1$, $\lambda = 1.1$, so $\beta = \ln 3/\ln 2.9 \approx 1.032$.

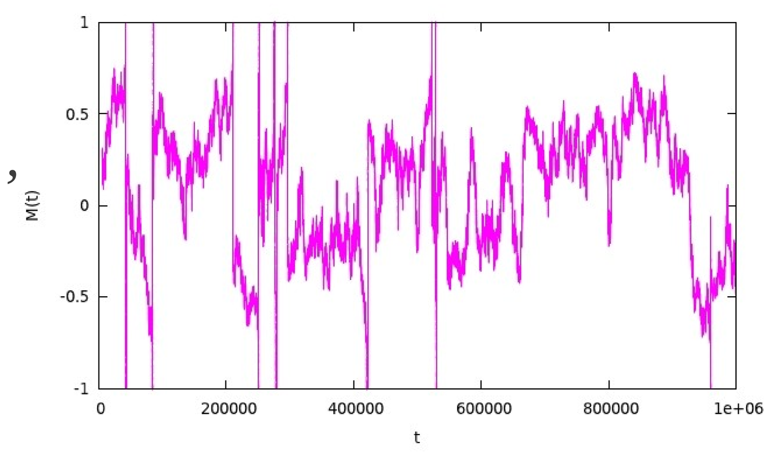

Fig. 9. Magnetization (logarithm of price) chart.

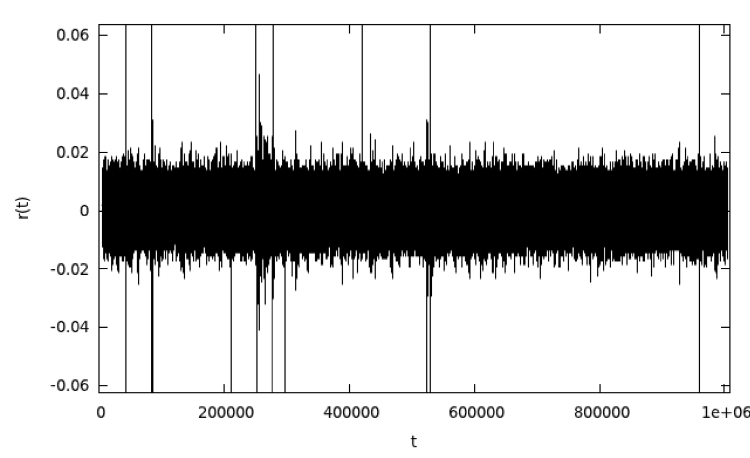

Fig. 1. Returns $r(t)$ in time t.

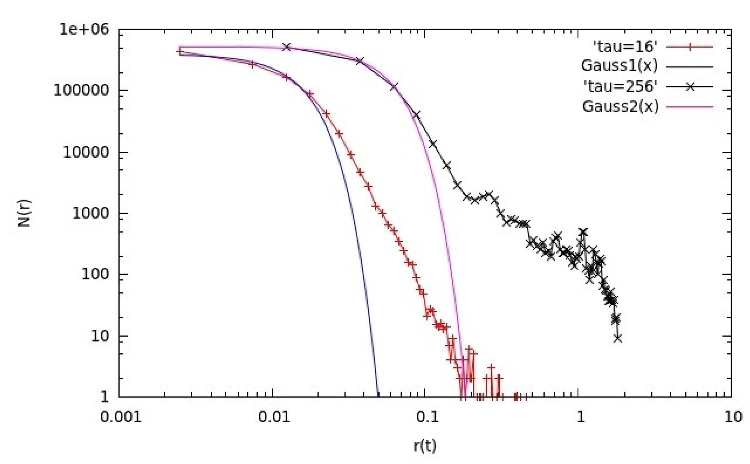

Fig. 2. Histograms of returns $r_\tau(t)$ for $\tau = 16$ (red) and $\tau = 256$ (black) with fitted Gaussians.

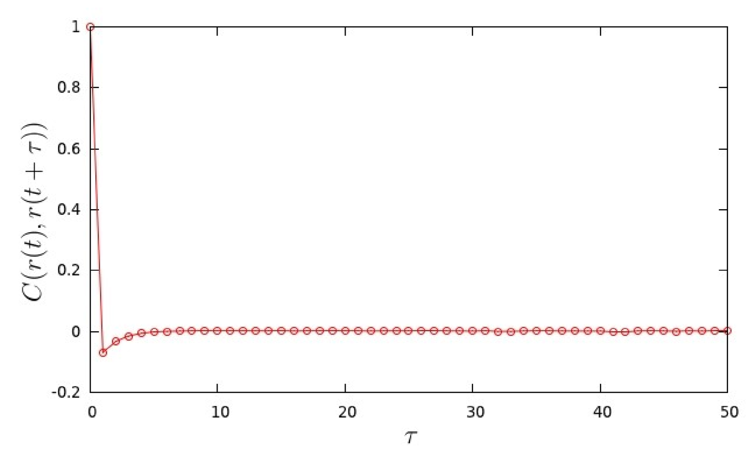

Fig. 3. Autocorrelation function of return r(t).

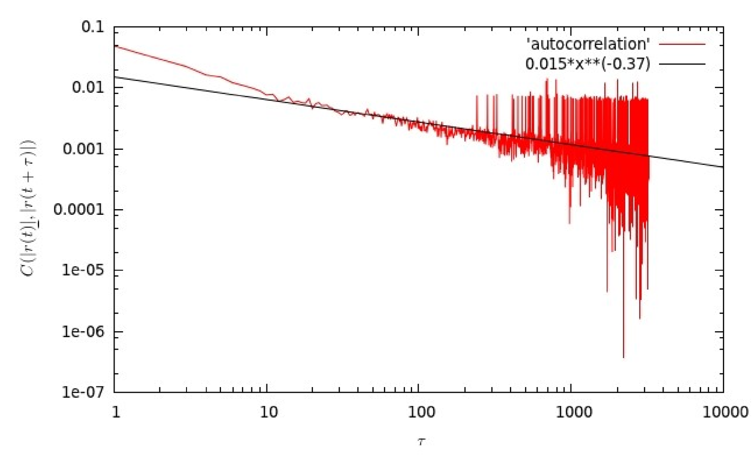

Fig. 4. Autocorrelation function of absolute return |r(t)| (red) with fitted power-law (black).

Bibliography

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