Catastrophic bifurcations on financial markets.
A Phenomenological approach

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Our works concerning the topic

[1] K. Kiyono, Z. Struzik, Y. Ymamoto:
Criticality and Phase Transitionin in Stock-Price Fluctuations,
PRL 96, 068701-1 – 068701-4 (2006).

[2] T. Werner, T. Gubiec, R. Kutner, D. Sornette:
Modeling of super-extreme events: An application to the hierarchical Weierstrass-Mandelbrot Continuous-Time Random Walk in Power-laws in real systems and beyond,
Eur. Phys. Special Topics, 2005, 27-52 (2012).

[3] T. Ciepliński, A. Dominiczak, R. Kutner: Short Comprehensive Report on the Non-Brownian Stochastic Dynamics at Financial and Commodity Markets, Acta Phys. Pol. A, 121, 24-27 (2012).

[4] M. Kozłowska, R. Kutner: Modern Rheology on a Stock Market: Fractional Dynamics of Indices, Acta Phys. Pol. A 118 (2010) 677.

Catastrophic (critical) bifurcation transition in a real life.

M. Scheffer et al., Nature 491 (2009), 53-59

In medicine: epileptic seizure and asthma attack.

In geophysics: earth quake, volcano eruption, abrupt shift in ocean circulation or in climate.

In ecosystem degradation: changes in states of coral reefs, colaps of vegetation in semi-arid ecosystem, a drastic decreasing of bee populations.

In finance: euro for Europe ?

Here, we study recent worldwide financial crisis

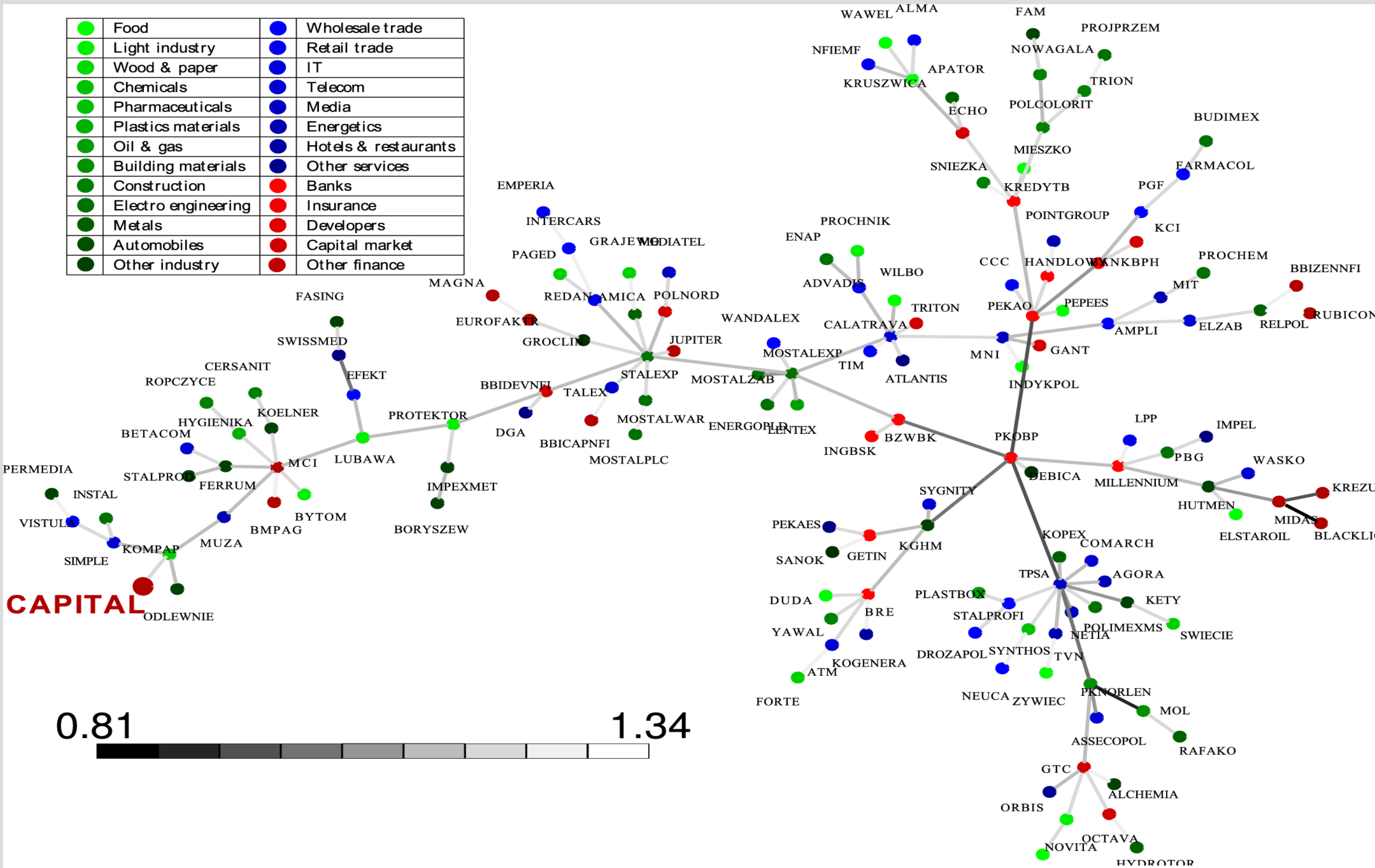
Our aim: **to present the concept of catastrophic bifurcation transition on financial markets in presence of superextreme events, without study of all scales and nonlinearity.**

First stage: **we are looking for the abrupt two-state transitions in a real world**

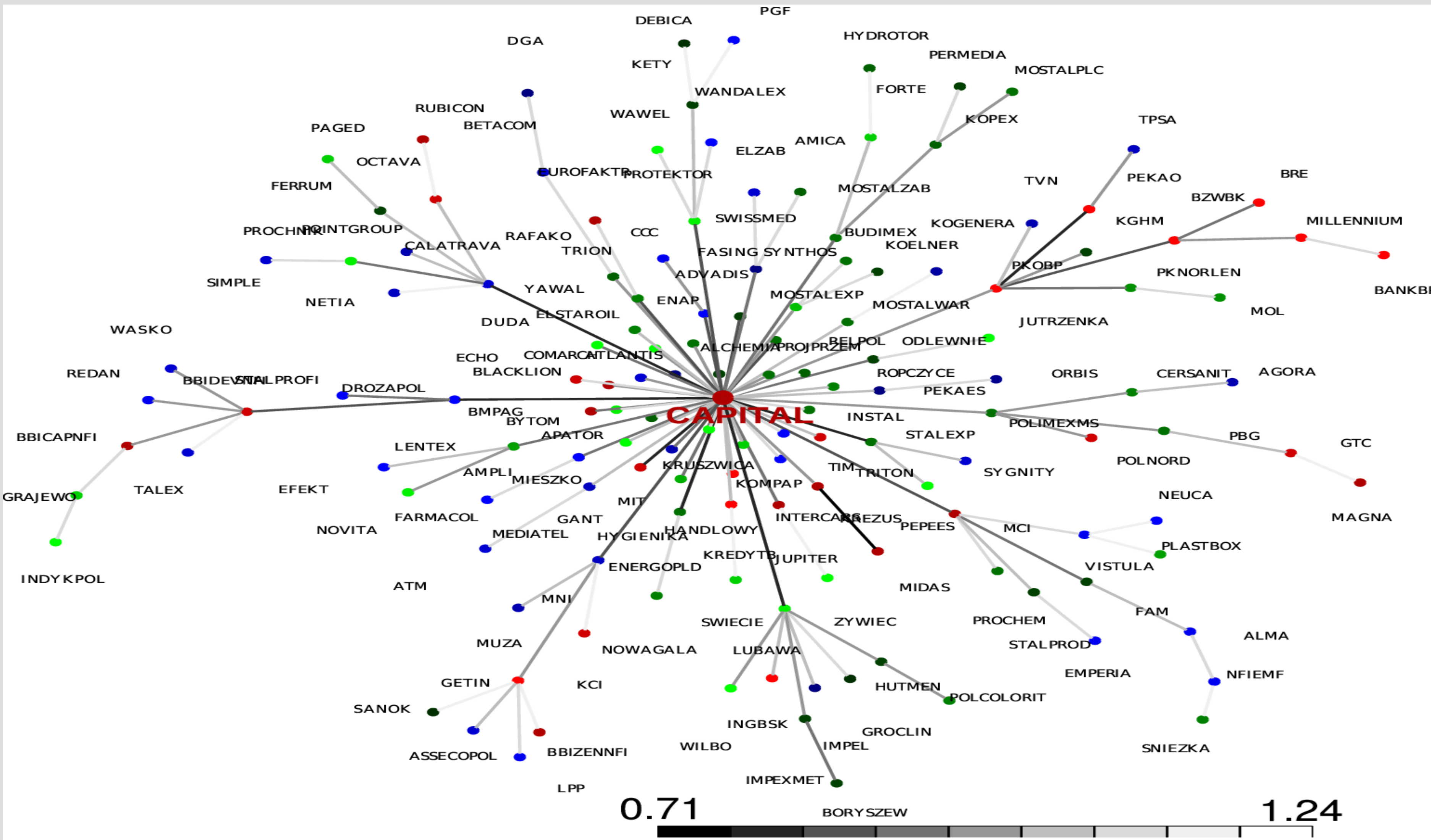
Second stage: **we are looking for the corresponding statistics before and after transition**

Third stage: **empirical verification of the hypothesis concernig the catastrophic bifurcation transition**

Branchy Minimal Spanning Tree for WIG
2005-01-03 - 2006-03-09
Does not include the crash region

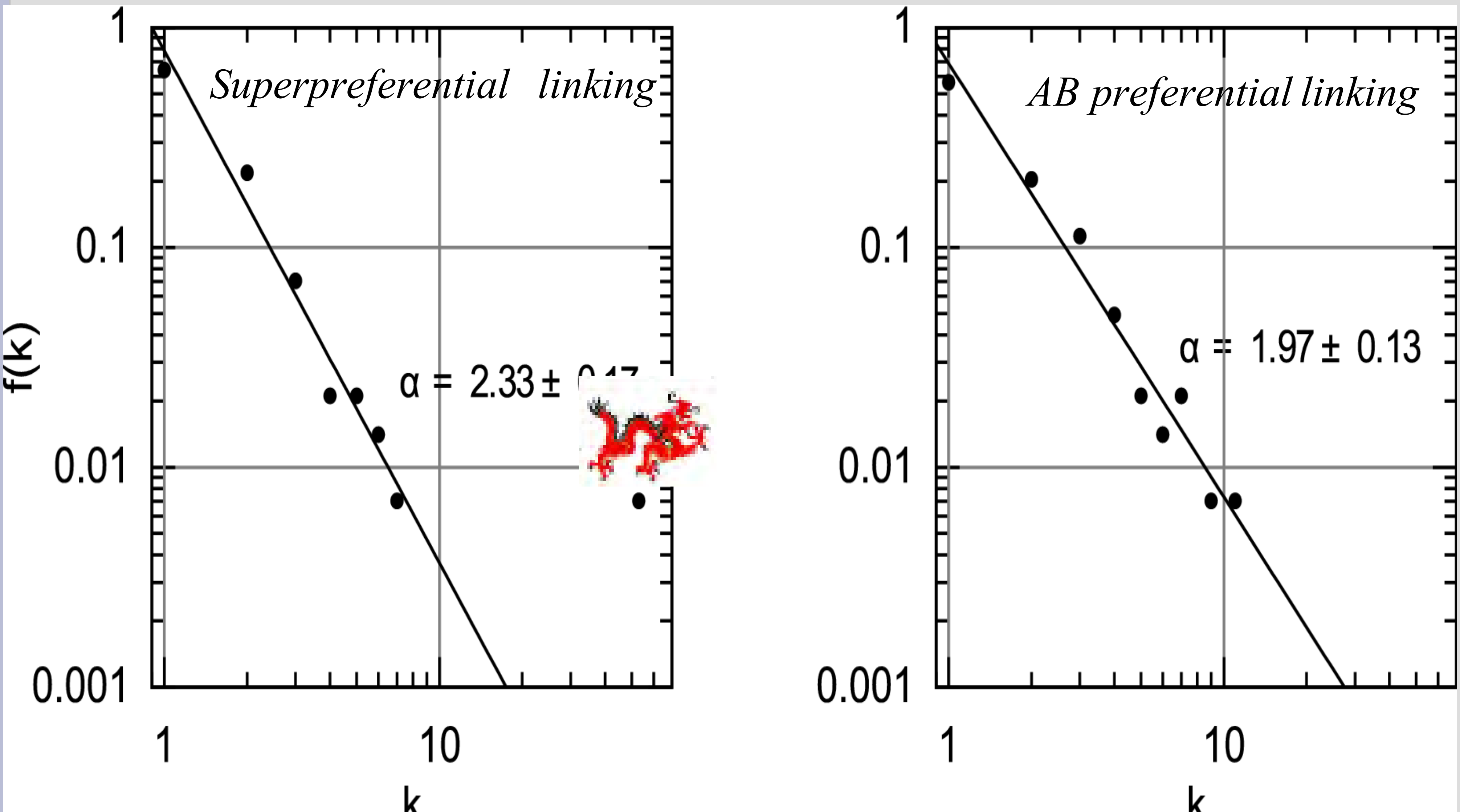


Centralized Minimal Spanning Tree for WIG
2007-06-01 - 2008-08-12
Includes the crash region



Distribution of degrees of graf MST vertices after and before 2006-05-05.

Abrupt structural transition from branchy to centralized MST

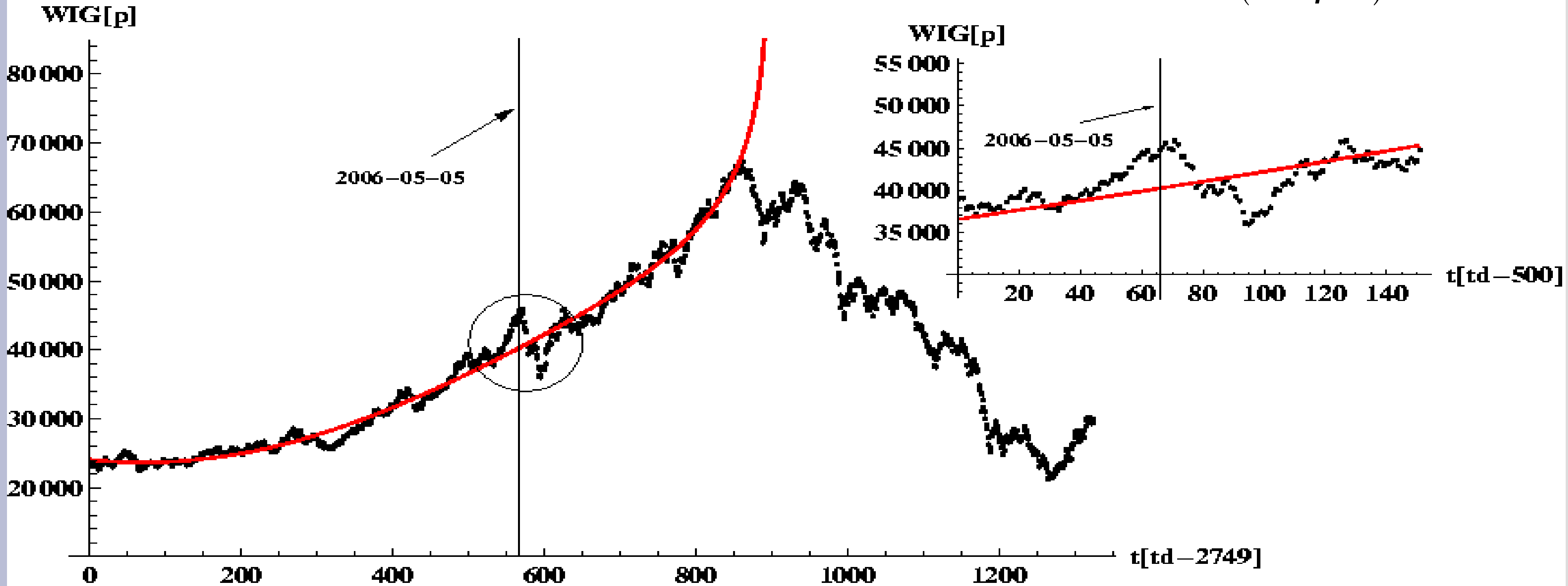


Index WIG of the Warsaw Stock Exchange (GPW) 2004-02-06 – 2007-07-06 - 2009-05-18.

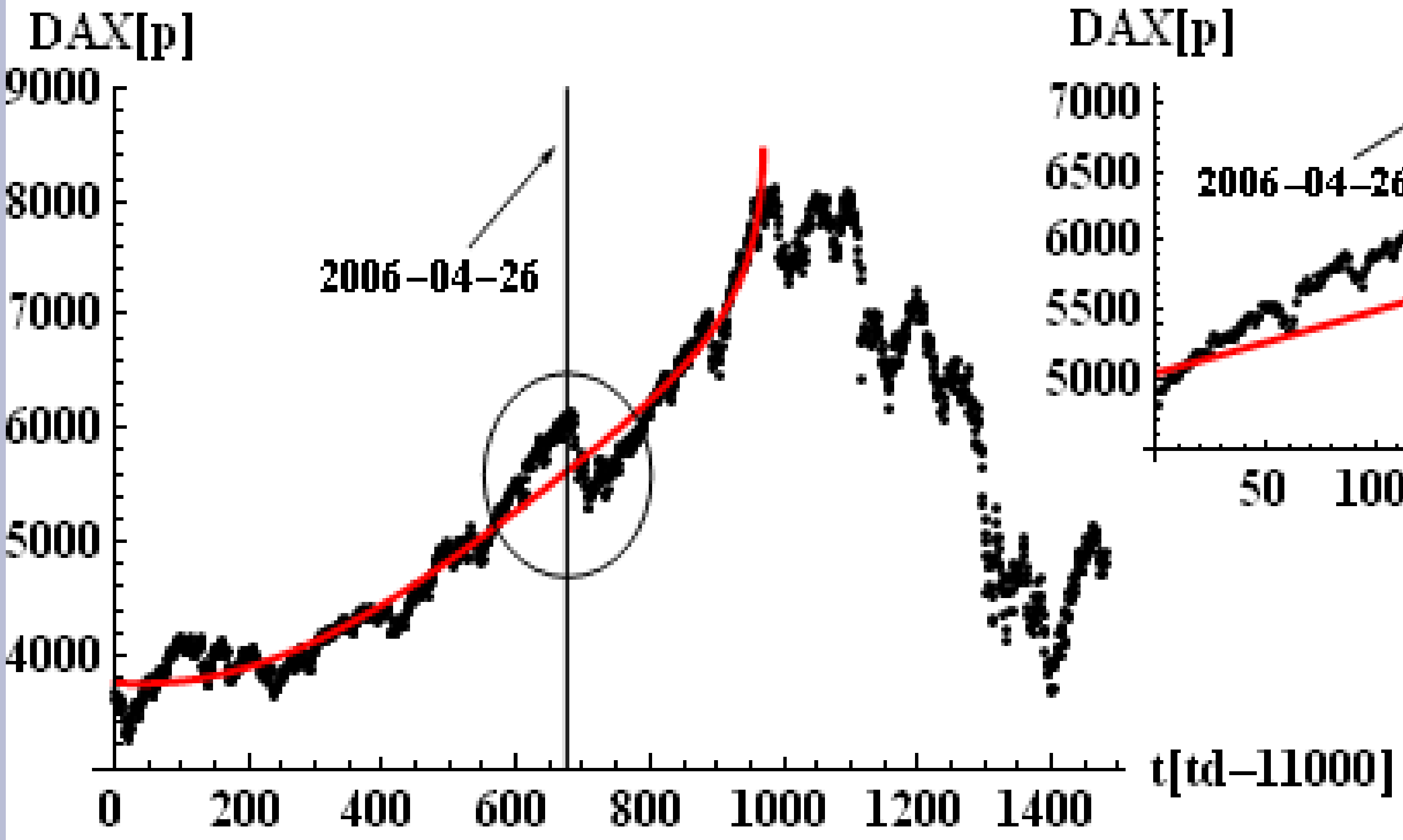
Here, we consider the lhs bubble

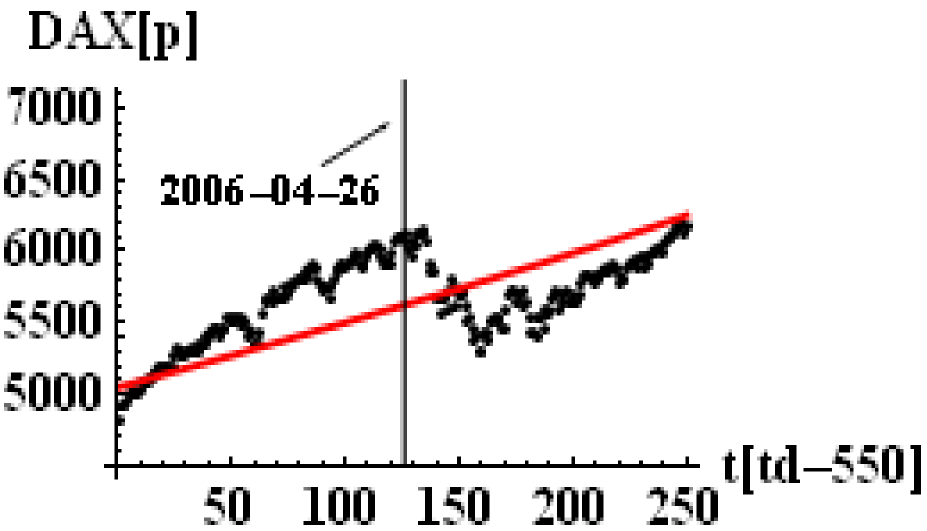
$$X(y)=\left(X(0)+A_1\right)E_{\alpha}\left(-\left(y/\tau\right)^{\alpha}\right)-A_1\cos(\omega\, y)\cos(\Delta\,\omega), \quad y=t_c-t$$

$$generalized\ Mittag-Lefflera\ exp.\ funct.: \quad E_{\beta}\left(-\left(y/\tau\right)^{\alpha}\right)=\sum_{n=0}^{\infty}\frac{\left(-\left(y/\tau\right)^{\alpha}\right)^{n}}{\Gamma(1+\beta\, n)}, \quad \beta=\alpha$$

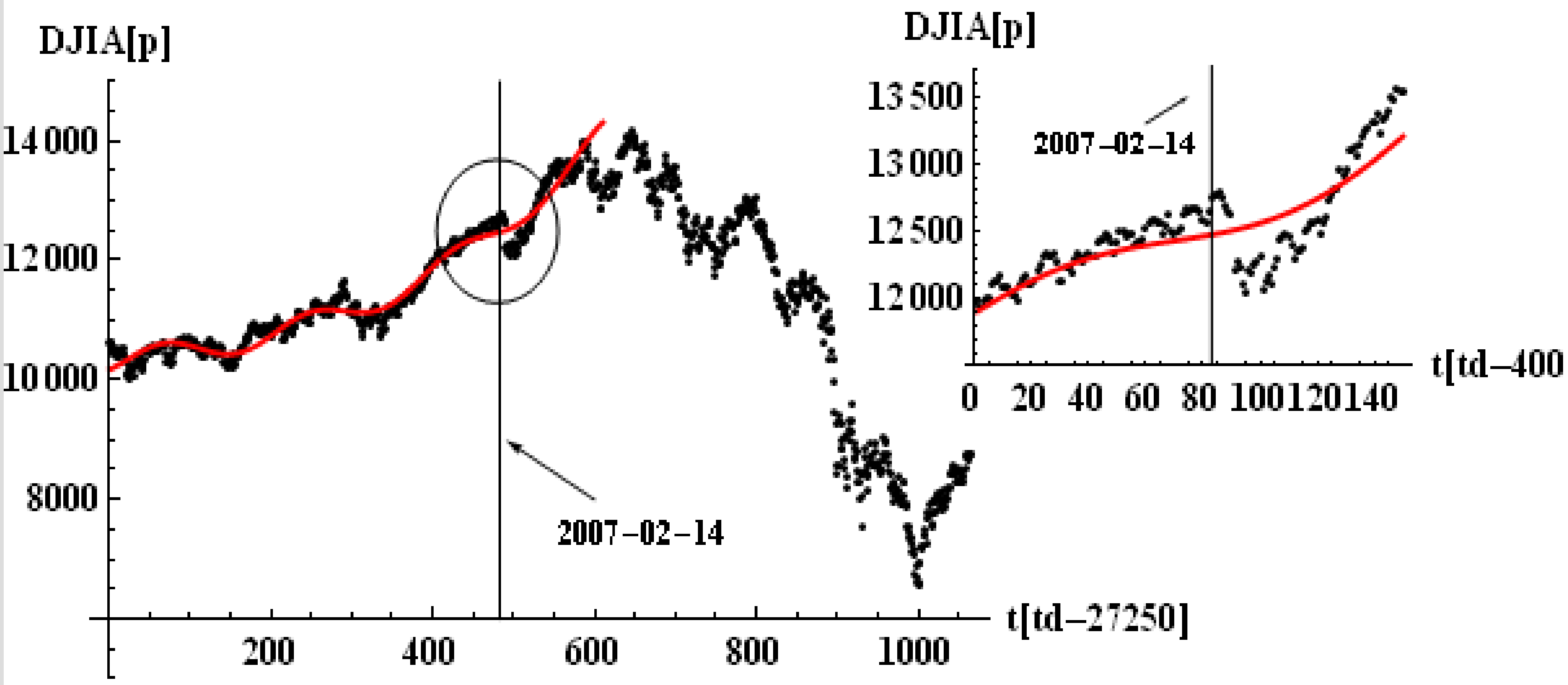


DAX: 2003-09-04 – 2007-07-13 - 2009-07-01.
Here, we consider the lhs bubble

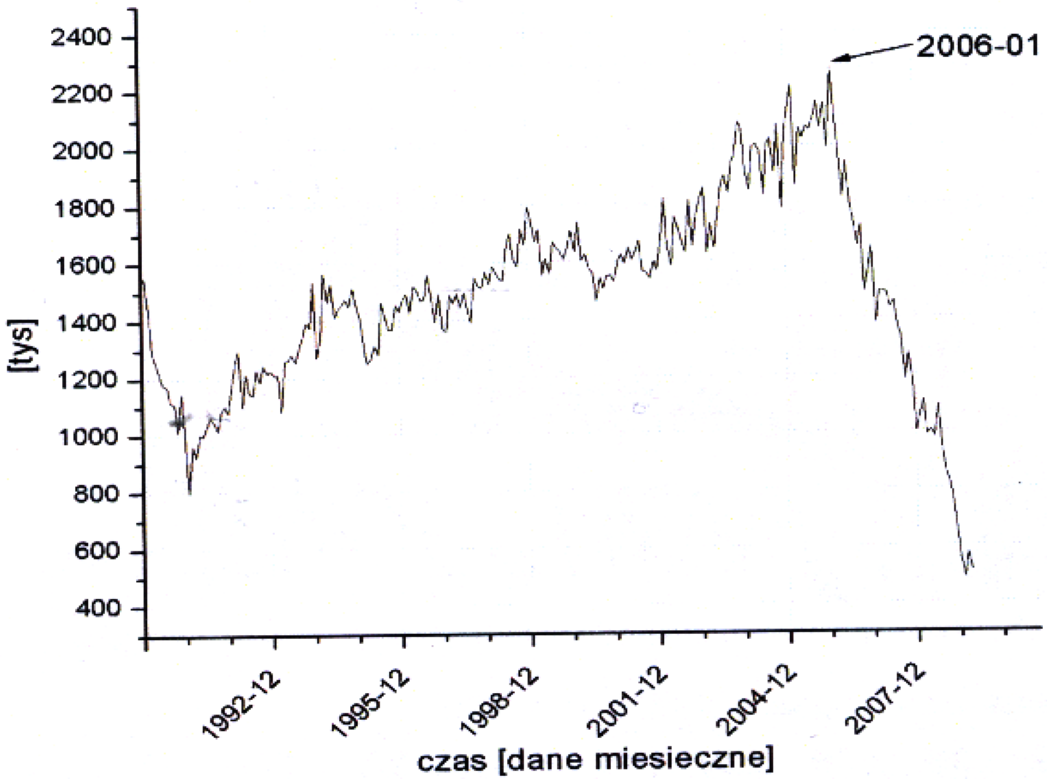




DJIA: 2005-03-16 – 2007-10-09 - 2009-06-09.
Here, we consider the lhs bubble

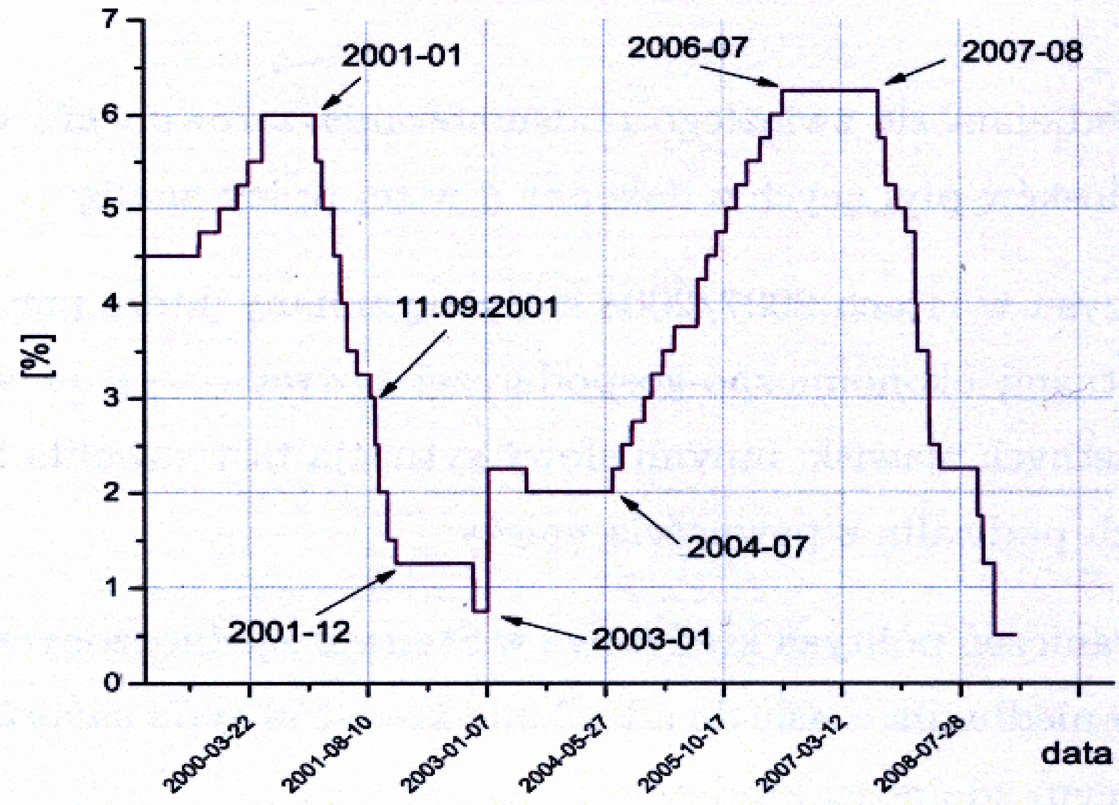


Beginning of new single-family house-buildings in US
1990-01 - 2009-03



Rysunek 2.9: Rozpoczęte budowy nowych domów w USA w okresie 01.1990-03.2009. Zaznaczone maksimum przypadające na styczeń 2006 wynosi 2273 tys. domów. Źródło danych: http://www.economagic.com, opracowanie własne.

FED reference rate
1999-01 - 2009-03



Rysunek 5.1: Stopy procentowe FED w okresie od początku 1999 do połowy marca 2009 roku. Na wykres naniesiono daty charakterystycznych punktów oraz datę zamachu terrorystycznego z 2001 roku. Źródło: http://www.economagic.com

Detrended signal and stochastic dynamics

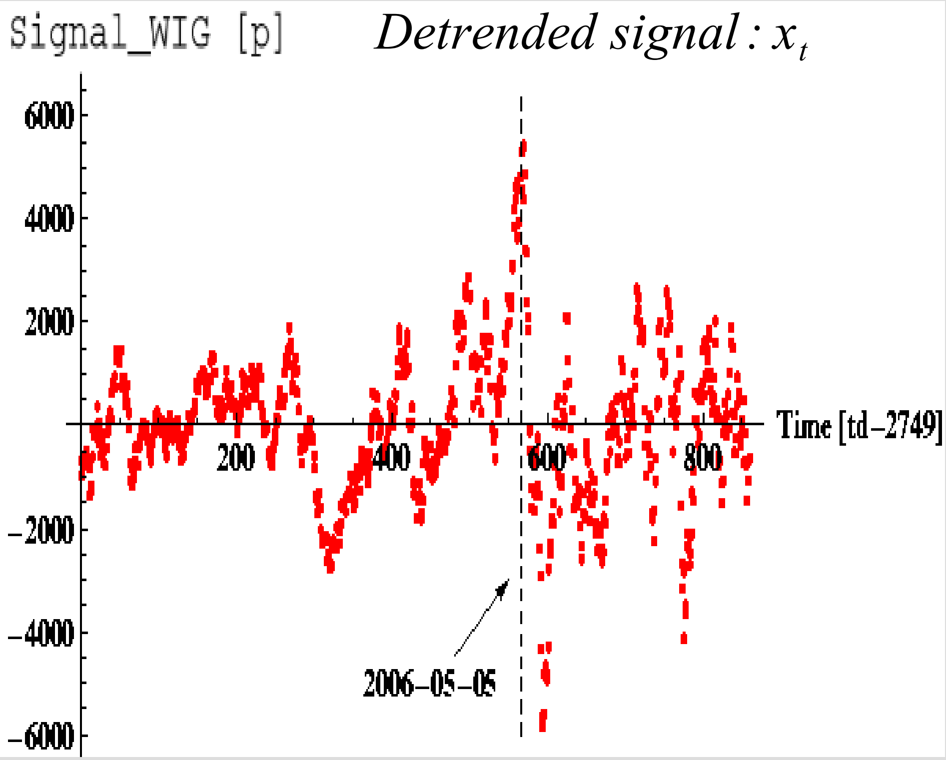
$$x_{t+1} - x_t = f(x_t; P) + \eta_t$$

x_t: *time-dependent daily signal*

P: *control parameter*

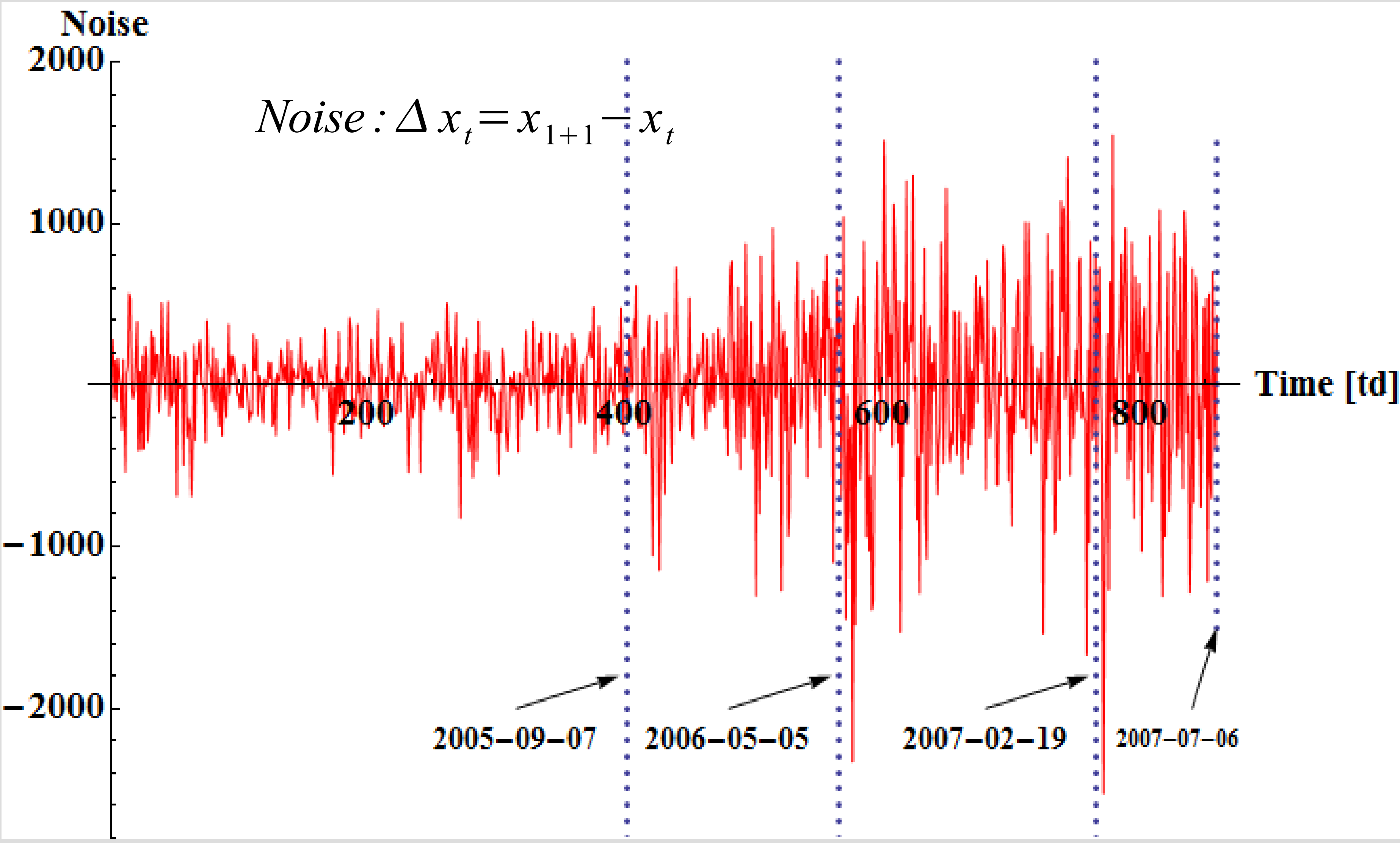
η_t: δ-*correlated* $(0, \sigma^2)$ *rand. var.*

$$f(x_t; P) = ?$$

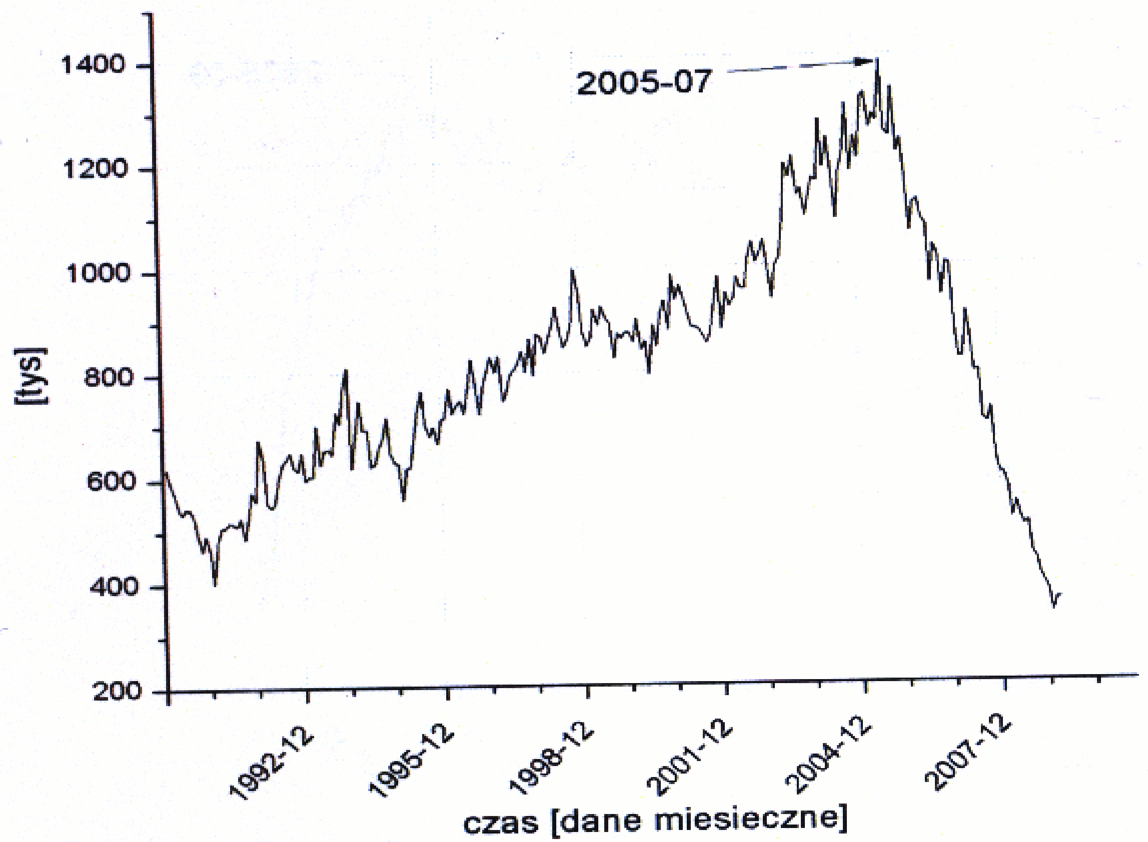


$1[td] \equiv 1991-04-16$ *the beginning of Warsaw Stock Exchange*

The noise of detrended signal WIG 2004.02.06 - 2007.07.06

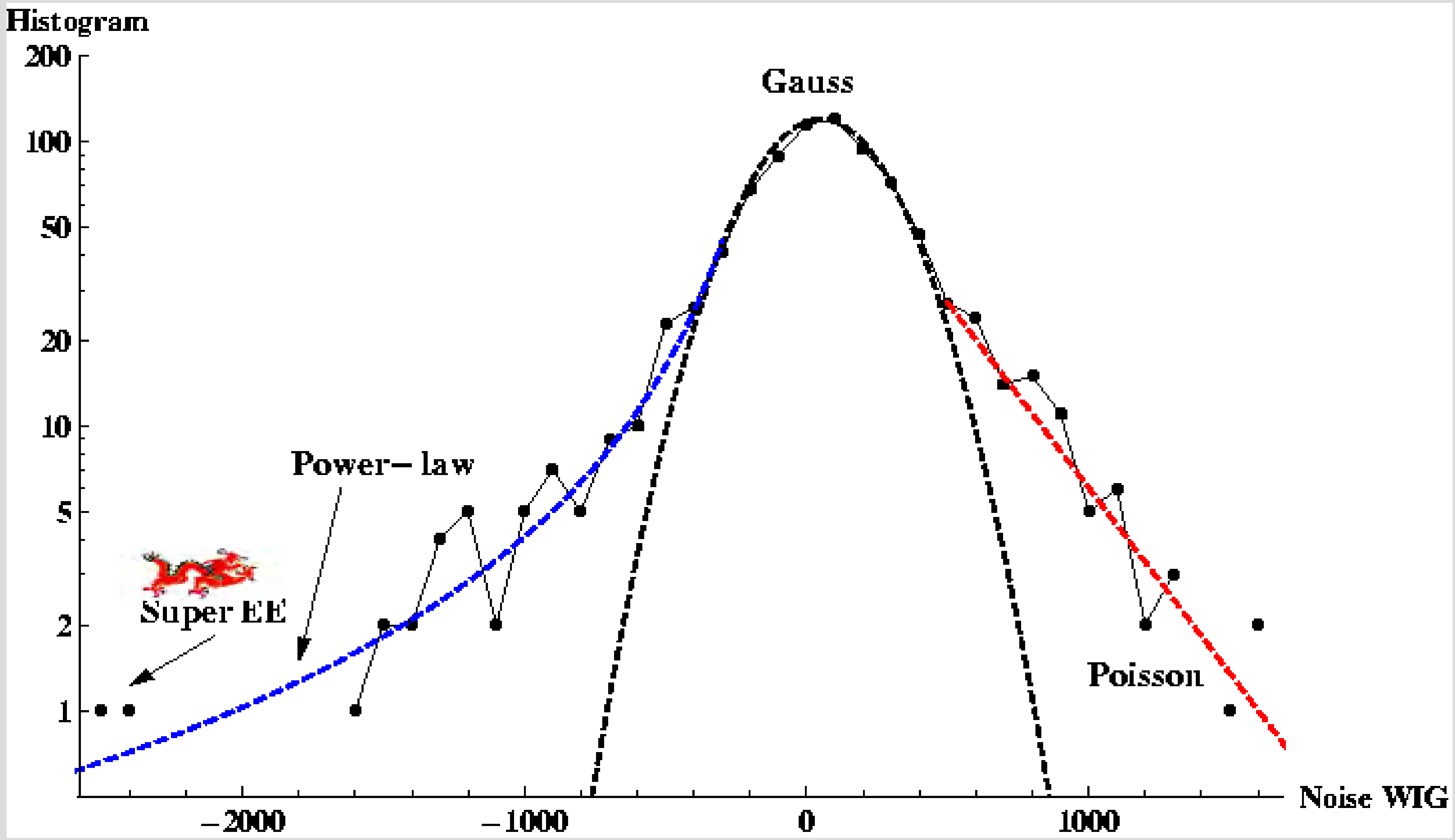


Sale of new single-family houses in US
1990-01 - 2009-03

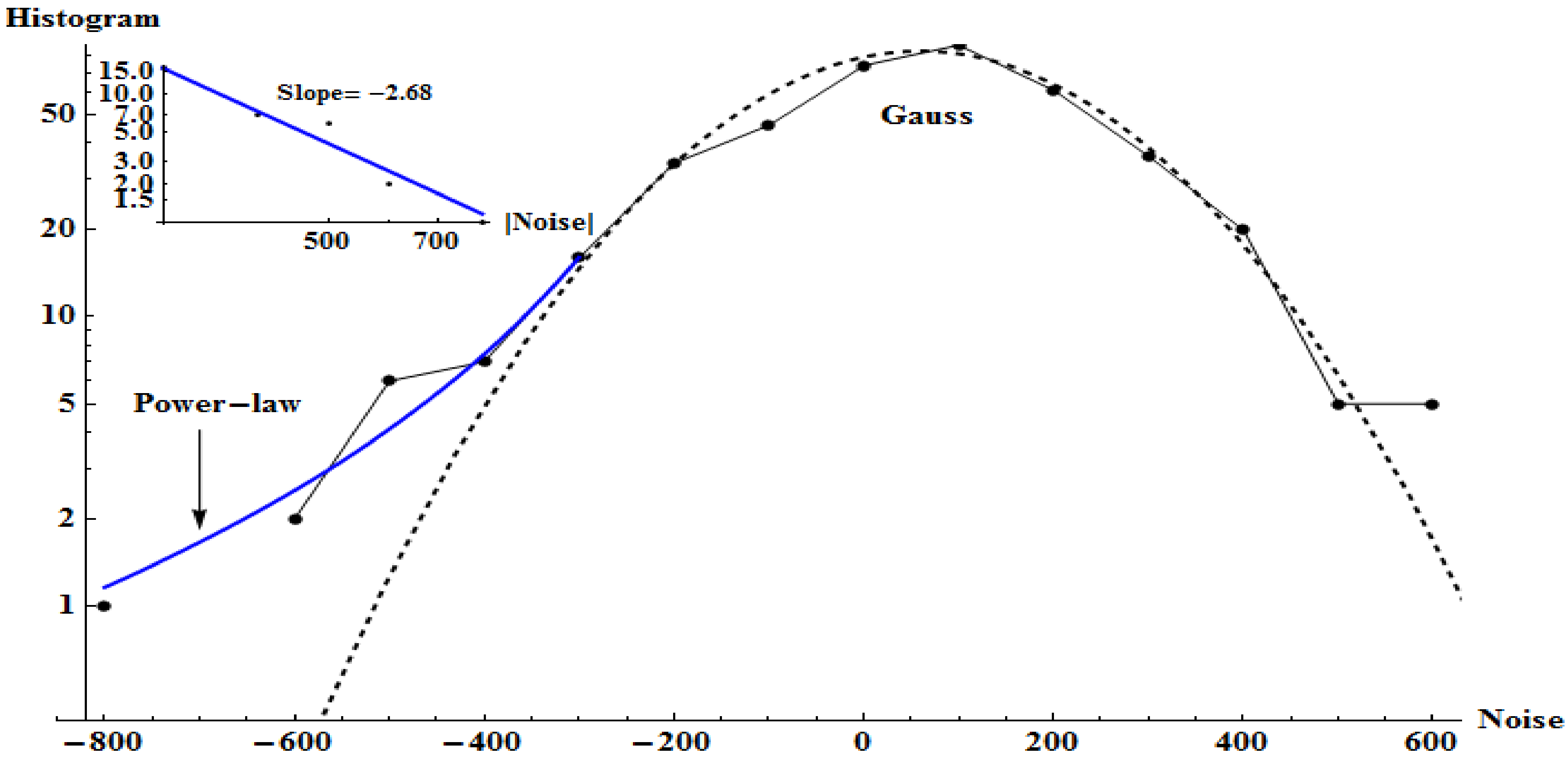


Rysunek 2.8: Ilość sprzedanych nowych domów jednorodzinnych w USA w okresie 01.1990-03.2009. Źródło danych: http://www.economagic.com, opracowanie własne.

Histogram of WIG's noise (decrements))
2004.02.06 - 2007.07.06

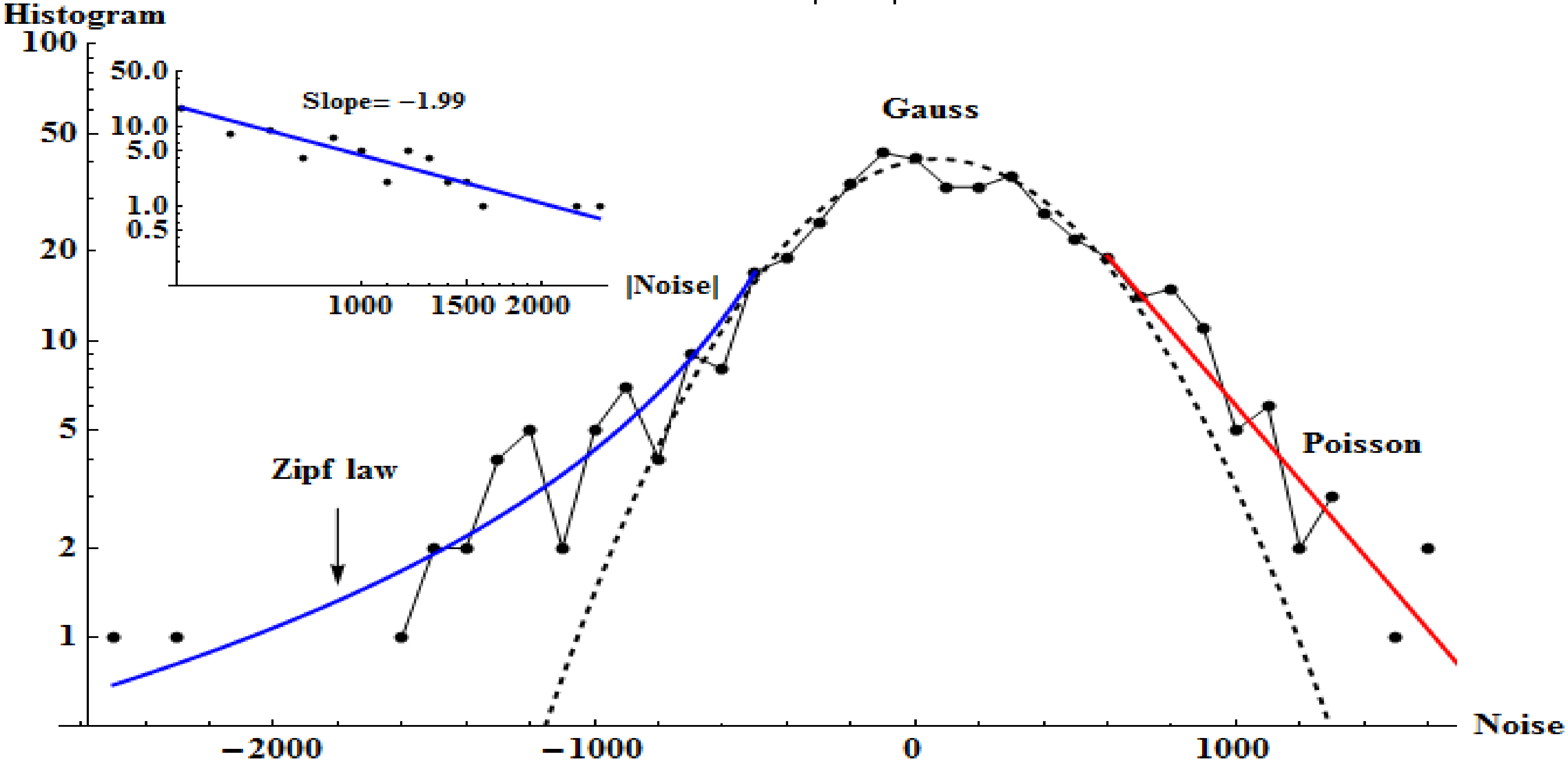


Mainly Gaussian histogram of WIG's noise: 2004.02.06 - 2005.09.07

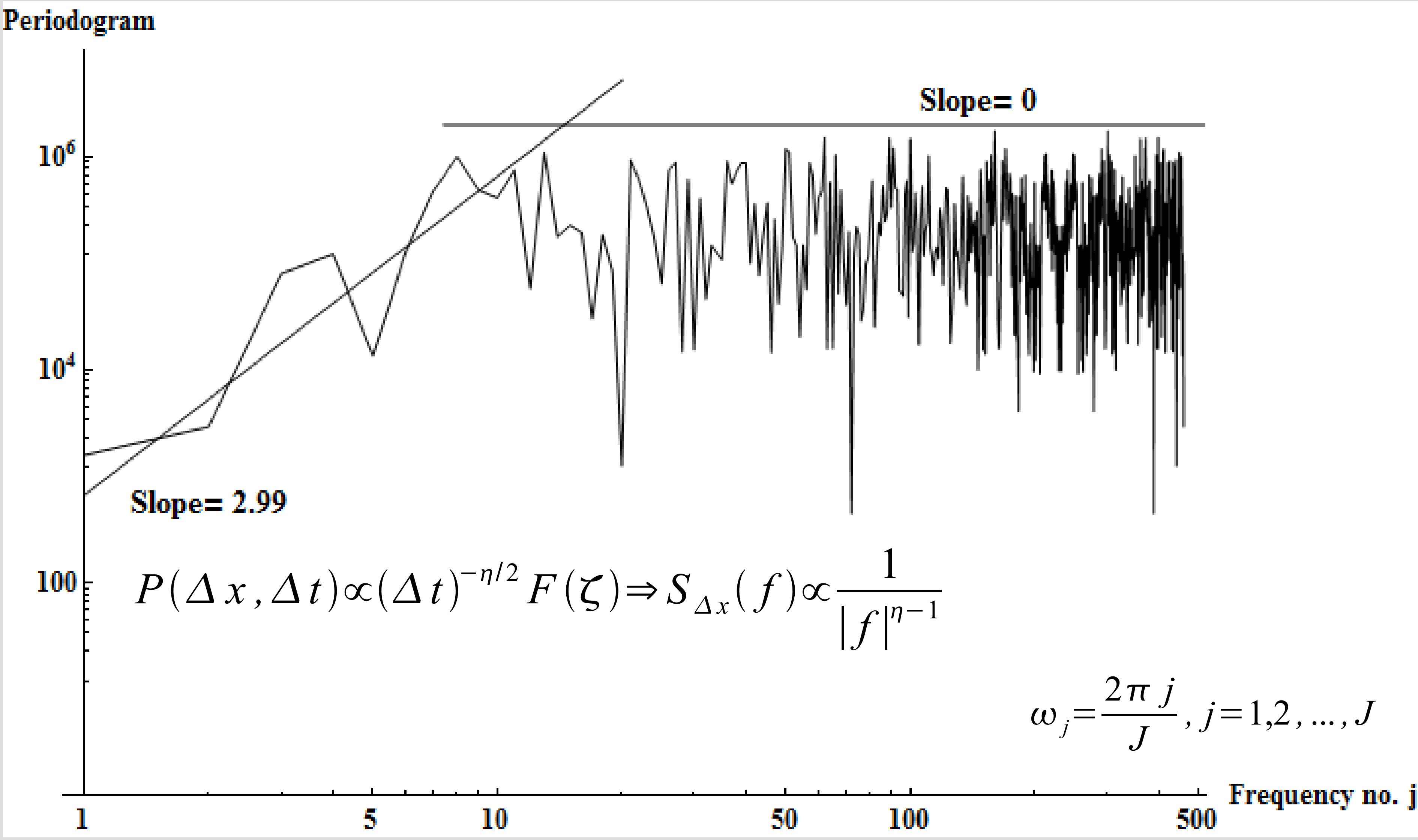


Asymmetric histogram of WIG's noise: 2005.09.08 - 2007.07.06

$$P(\Delta x, \Delta t) \propto (\Delta t)^{-\eta/2} F(\zeta) \Rightarrow P(\Delta x) \propto \frac{1}{|\Delta x|^{1-2/\eta}}, \quad \zeta = |\Delta x| / (\Delta t)^{\eta/2}, \eta = -2.02$$

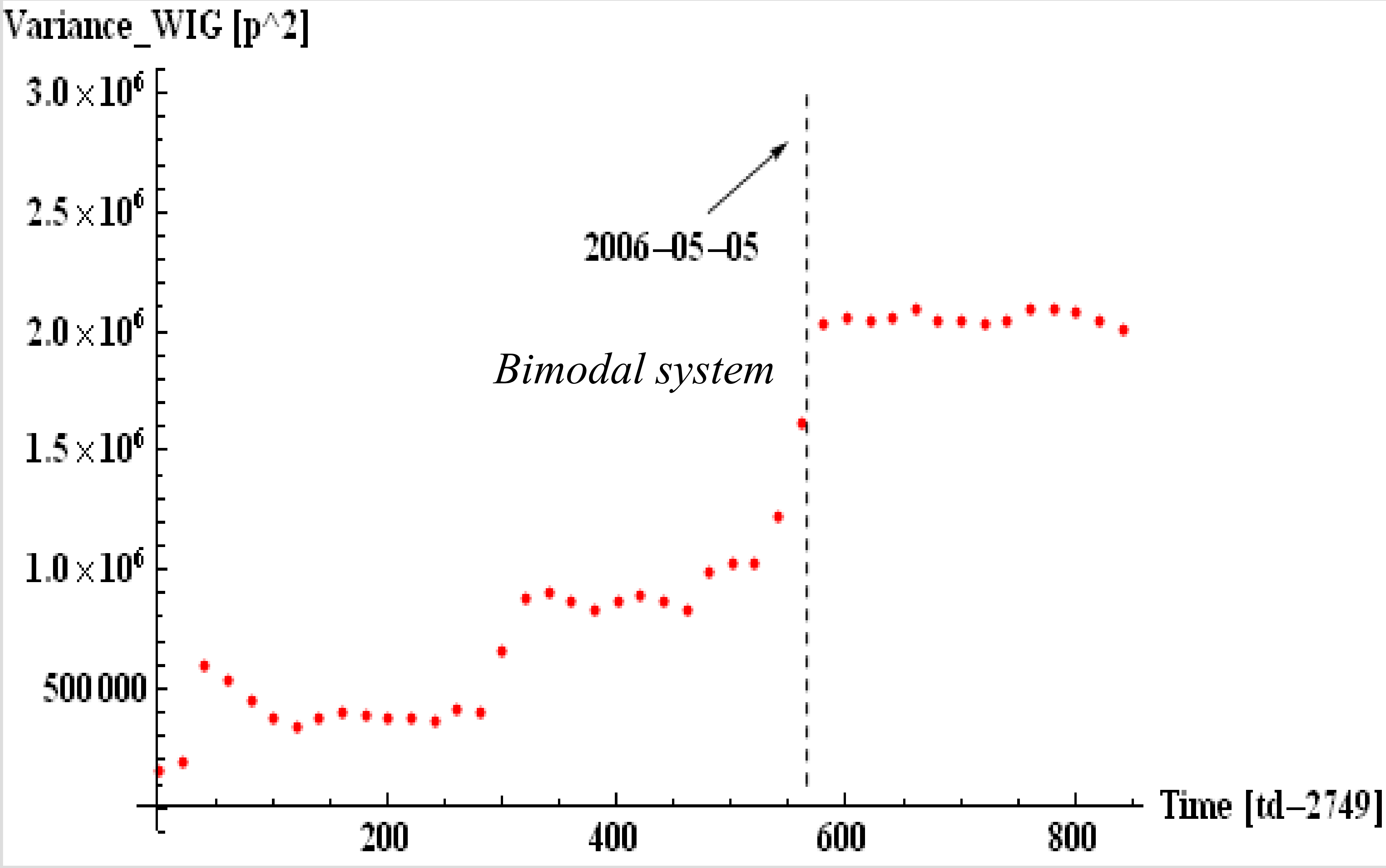


Periodogram of WIG's noise: 2005.09.08 - 2007.07.06



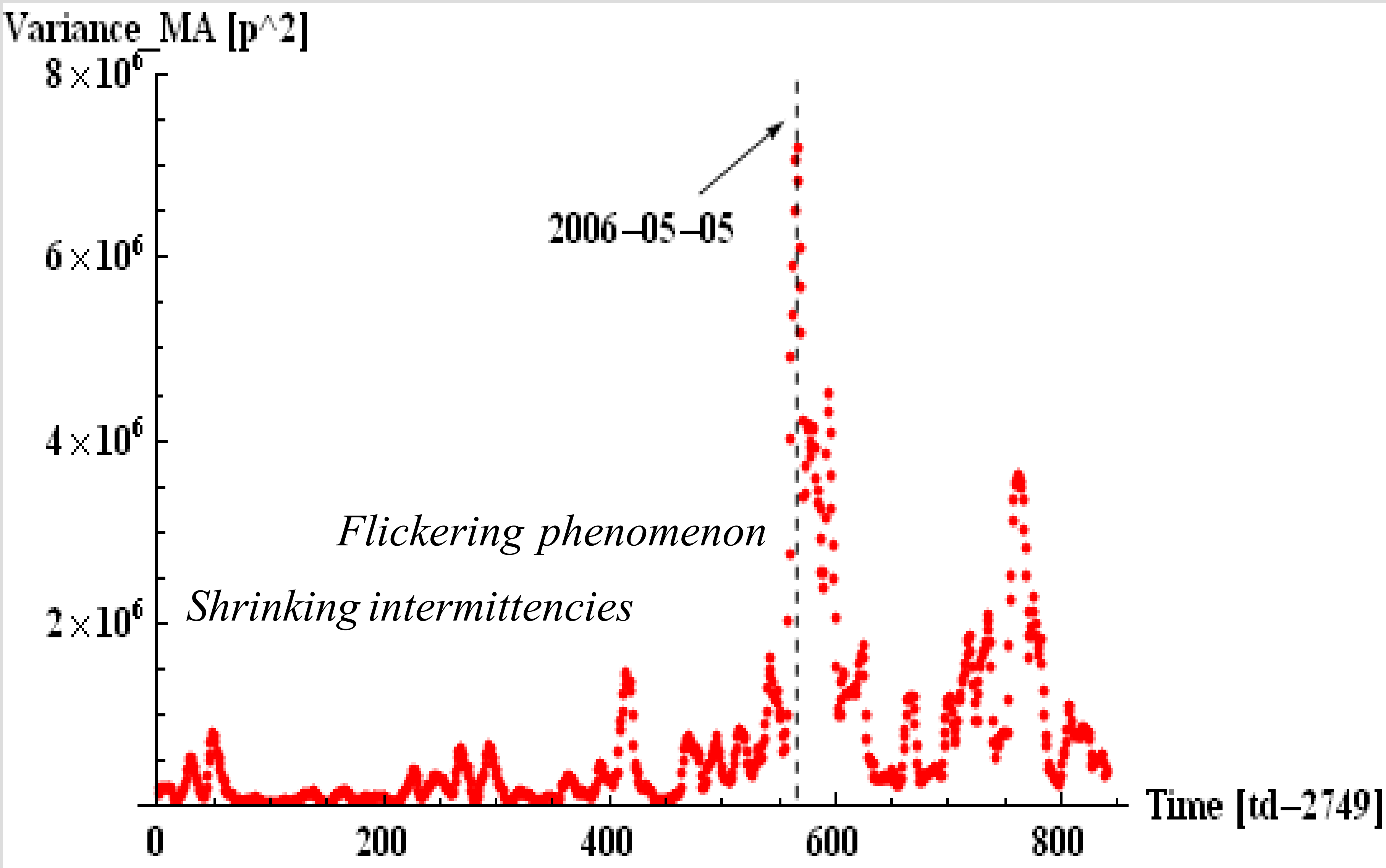
WIG: accumulative variance.

Bimodal situation



WIG: variance (monthly sample average).

Definition of threshold by spike



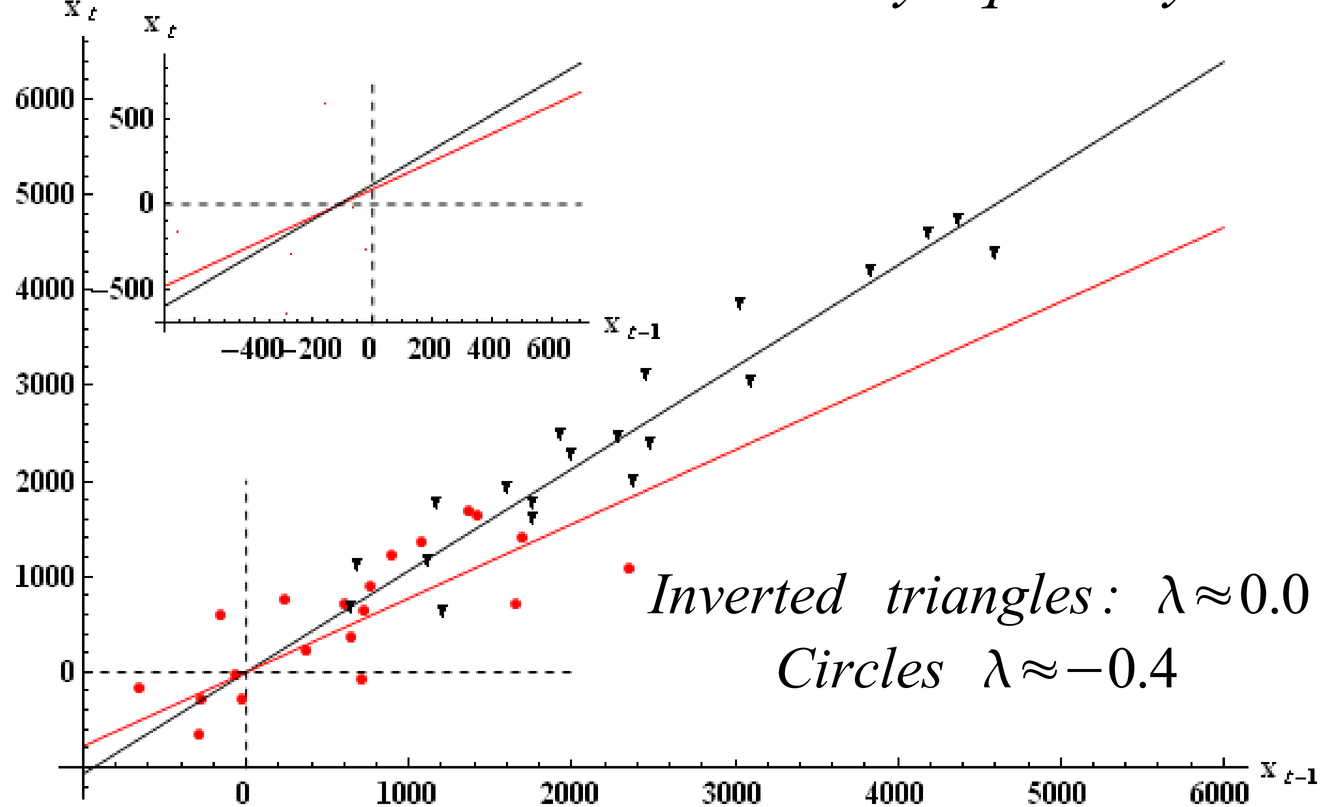
Detrended successive signals for WIG.
Two time scales:
(i) daily t (fast) one and (ii) monthly (slow) one.
Too large data dispersion for the nonlinear analysis

$$x_t = AR(1)\,x_{t-1} + b + \eta_{t-1}, \quad AR(1) = 1+\lambda, \quad b = -\lambda\, x_1^*,$$

$$ACF(1) = \frac{Cov(x_t, x_{t\pm 1})}{Var(x_t)} = 1+\lambda = AR(1)$$

$AR(1), ACF(1), \lambda:$ *slowly varying*

λ *is our key quantity*



Inverted triangles: $\lambda \approx 0.0$

Circles $\lambda \approx -0.4$

Application of catastrophe theory to financial markets.

Catastrophic or critical slowing down

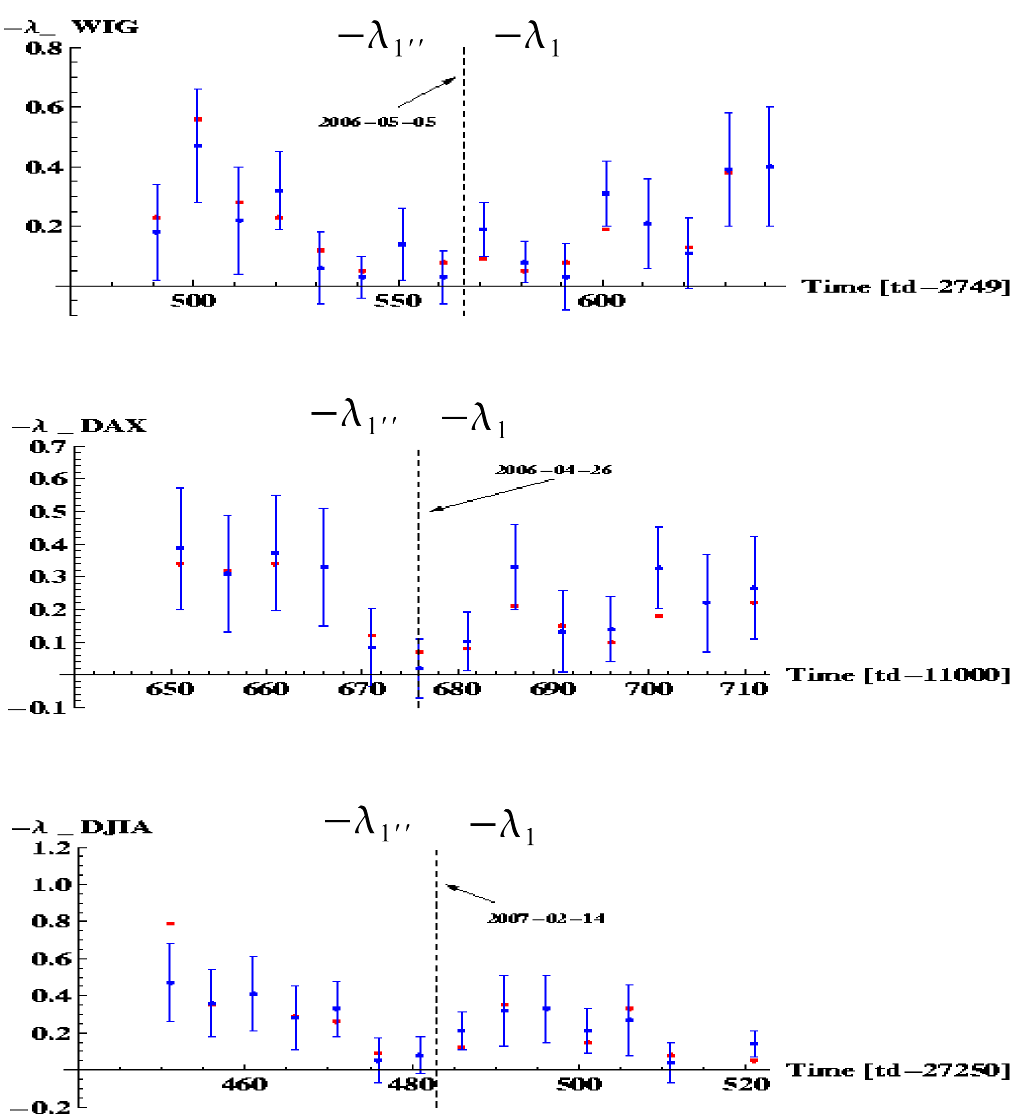
$$Blue: \ -\lambda = 1 - AR(1)$$
$$Red: \ -\lambda = 1 - ACF(1)$$

$$-\lambda \quad smile:$$

λ *is negative below threshold*

λ *reaches zero at threshold*

λ *after threshold is negative*



Application of catastrophe theory to financial markets.
Hypothesis: *catastrophic bifurcation transition*
Complete approach requires nonlinear analysis

Stochastic dynamics

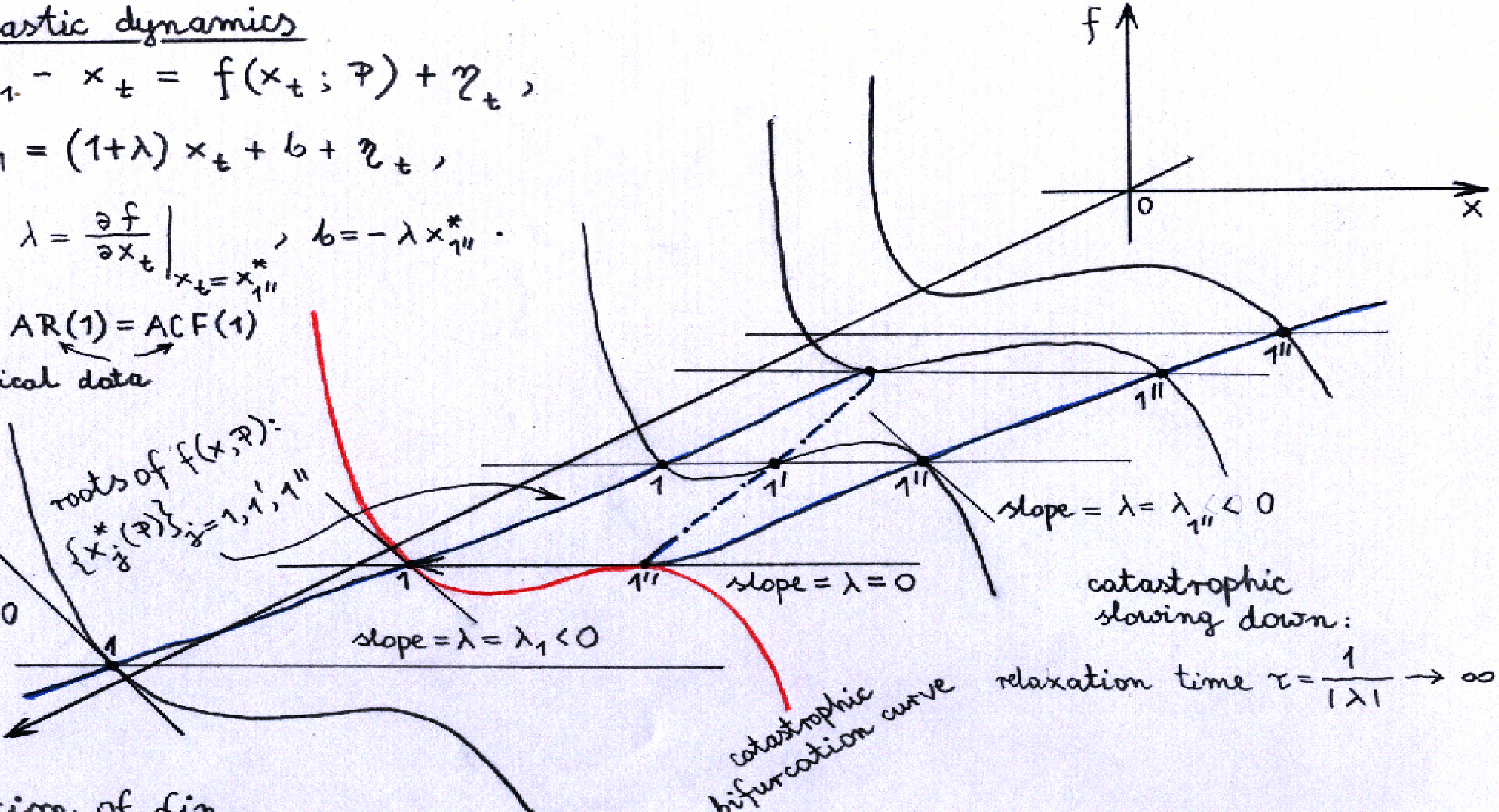
(1) $$x_{t+1} - x_t = f(x_t; \mathcal{P}) + \eta_t ,$$

Lin.: $$x_{t+1} = (1+\lambda)\, x_t + b + \eta_t ,$$

(2) $$\lambda = \left.\frac{\partial f}{\partial x_t}\right|_{x_t = x^*_{1''}} , \quad b = -\lambda x^*_{1''} .$$

(3) $$1+\lambda = AR(1) = ACF(1)$$

from empirical data



catastrophic slowing down:

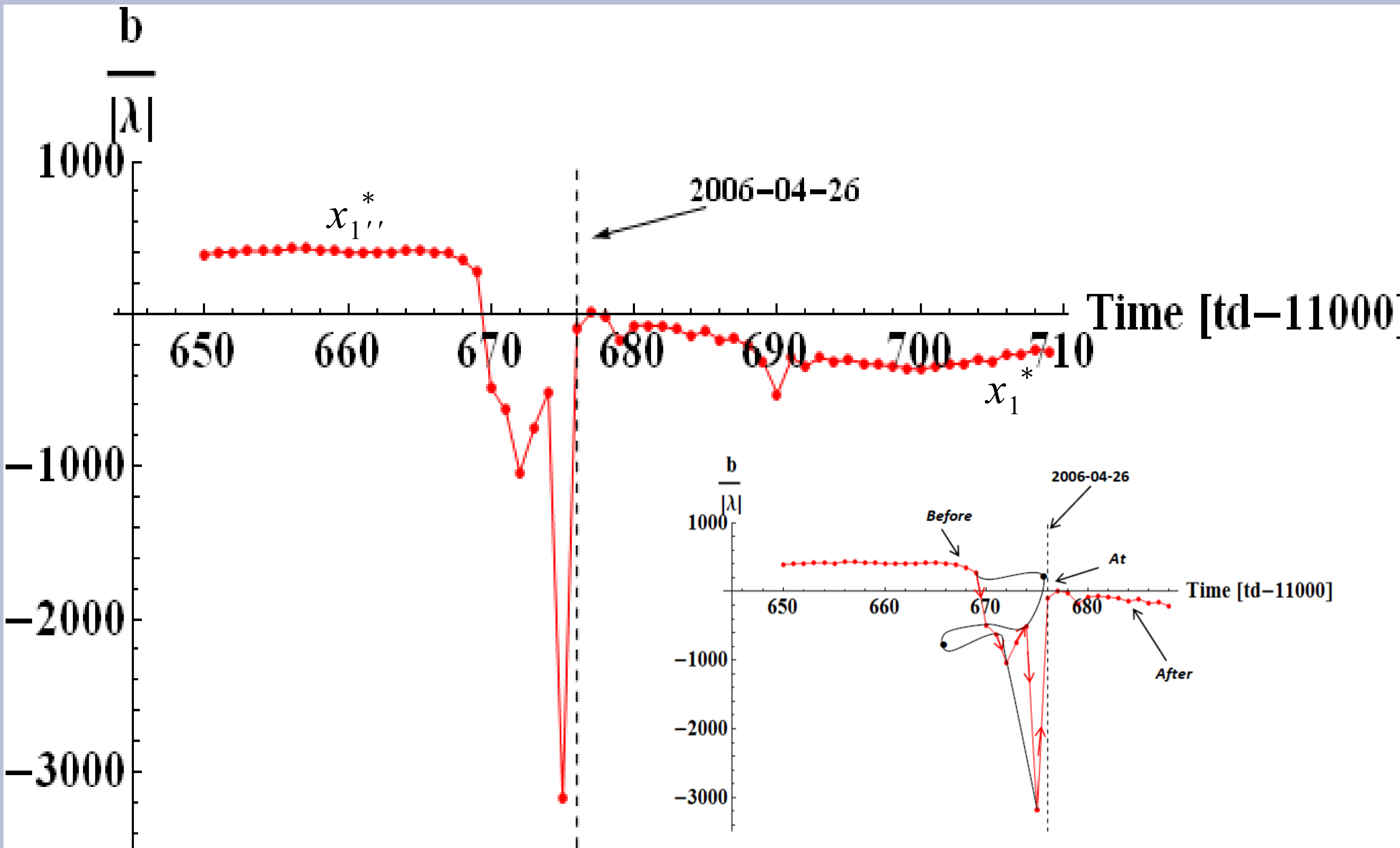
relaxation time $\tau = \frac{1}{|\lambda|} \to \infty$

Solution of Lin.

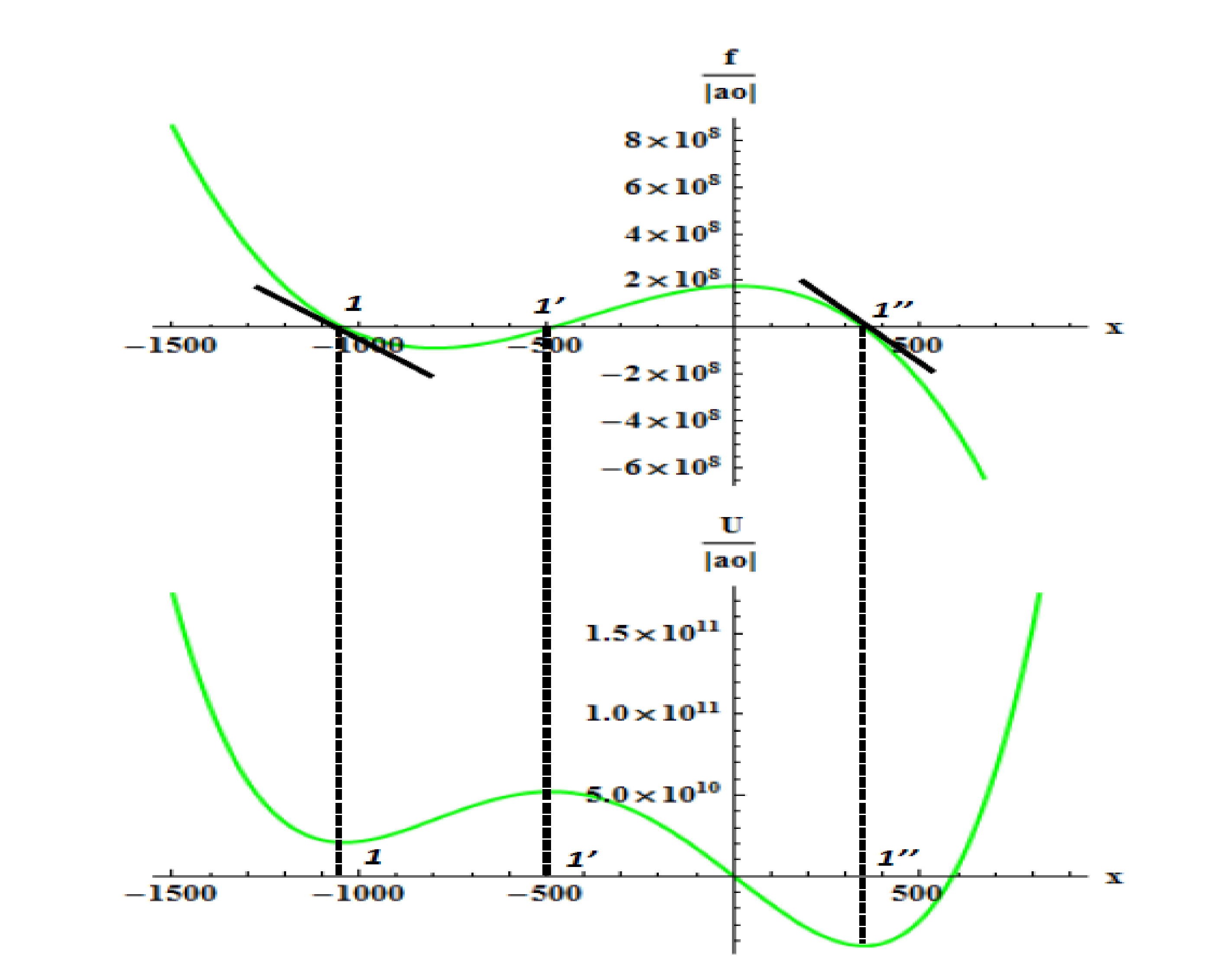
(4) $$y_t = \exp(\lambda t)\left[y_0 + \int_0^t \eta_\tau \exp(-\lambda\tau)\, d\tau \right], \quad y_t = x_t - x^*_{1''} , \quad t = 0, 1, 2, \ldots ,$$

(5) $$\langle y_t \rangle = \exp(\lambda t)\, \langle y_0 \rangle .$$

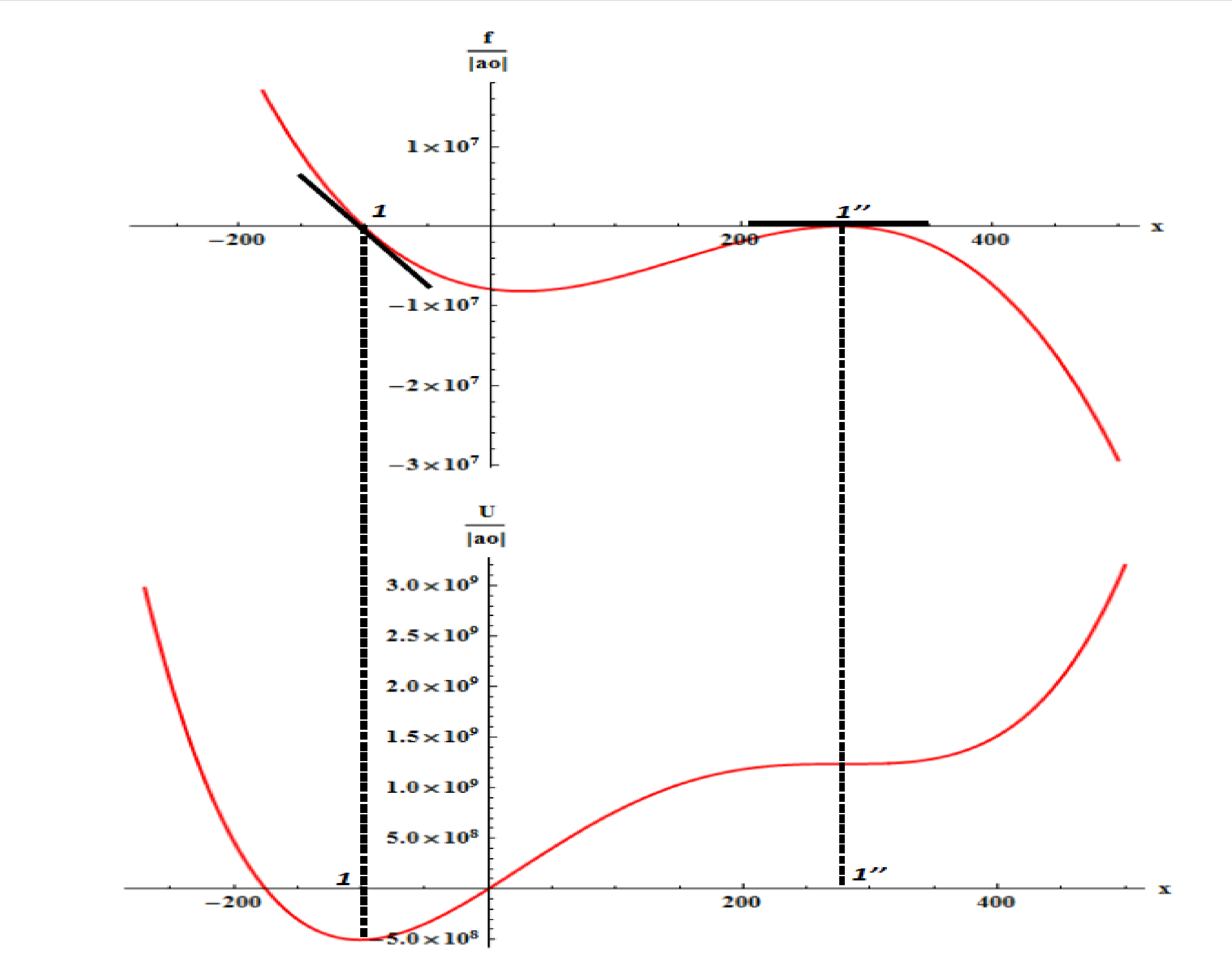
DAX: *Catastrophic & subcatastrophic bifurcation transitions*



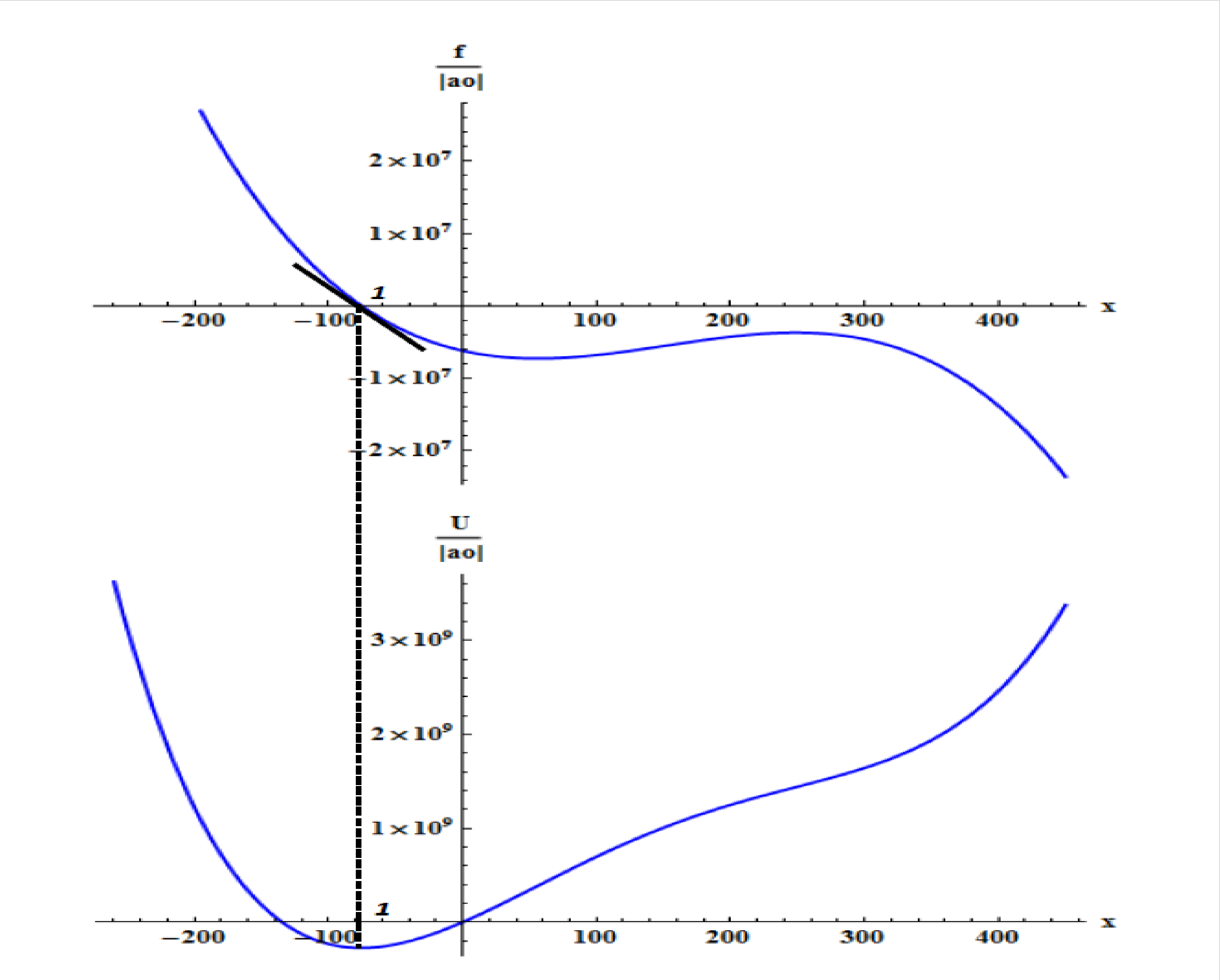
Possible force *f* and potential *U* before catastrophic bifurcation transition



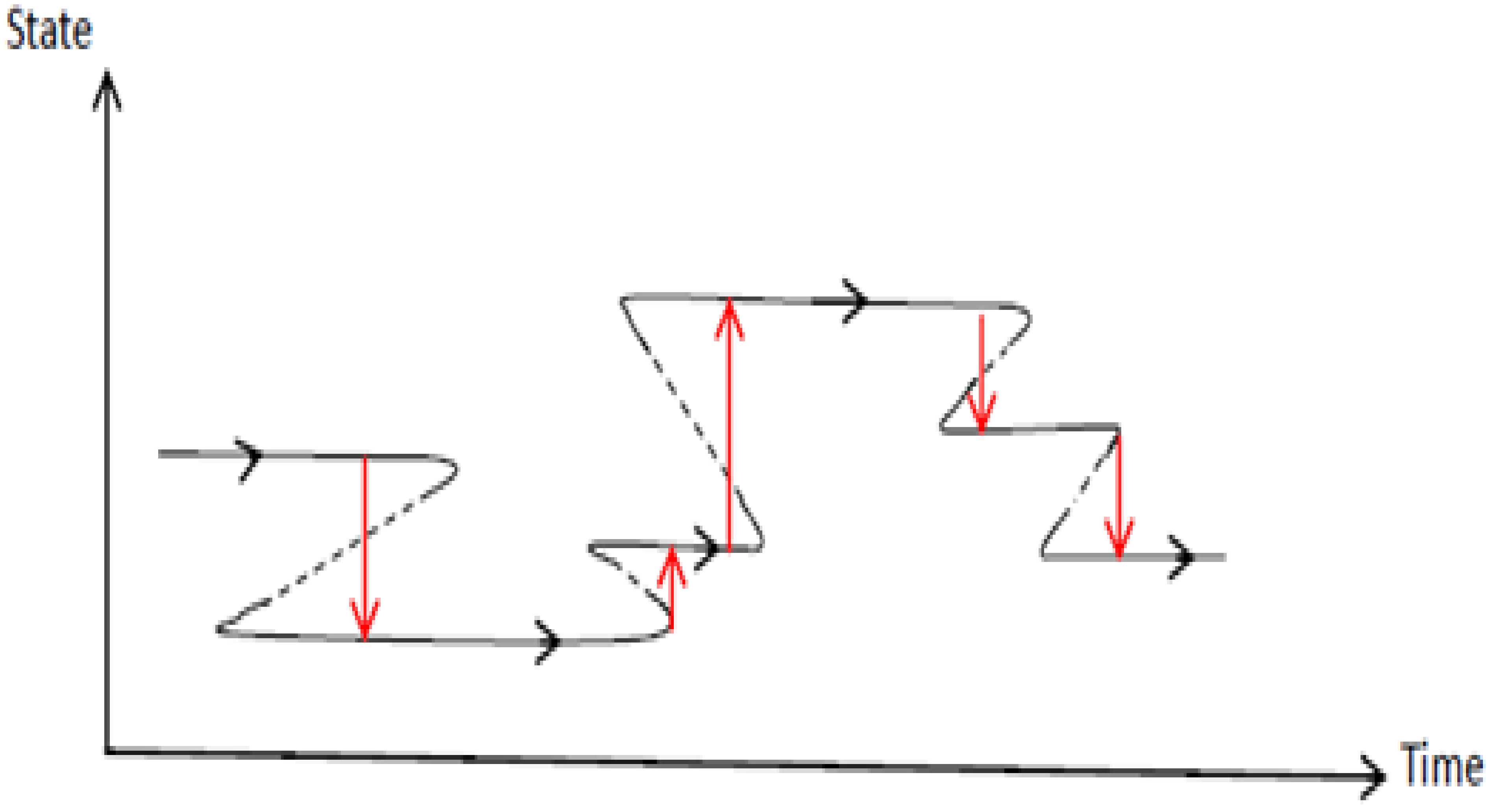
Possible force *f* and potential *U* at catastrophic bifurcation transition



Possible force *f* and potential *U* after catastrophic bifurcation transition



Schematic hypothetical scenario of stock market evolution generated by catastrophic bifurcation transitions



Podsumowanie

1) Symptomy katastroficznych bifurkacji na giełdach.

2) Konsekwencje katastroficznych bifurkacji.

3) Katastroficzne spowolnienie ('catastrophic slowing down').

4) Teoria skalowania.

5) Hipoteza prekursorów krachu.

6) Elementy scenariusza ewolucji giełd.

7) Zamierzenia badawcze.