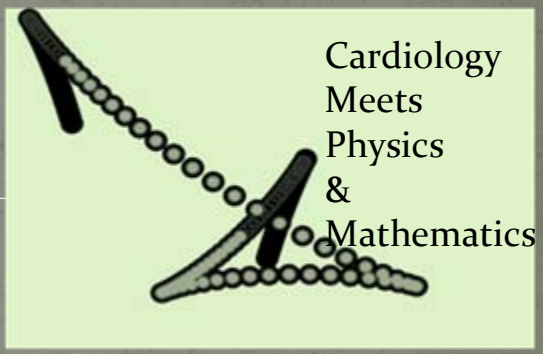
FENS Warszawa 25-27.XI.2010

5 Ogólnopolskie Sympozjum

Fizyka w Ekonomii i Naukach Społecznych

multifractal estimators of long range dependence

Multifraktlane estymatory Hursta



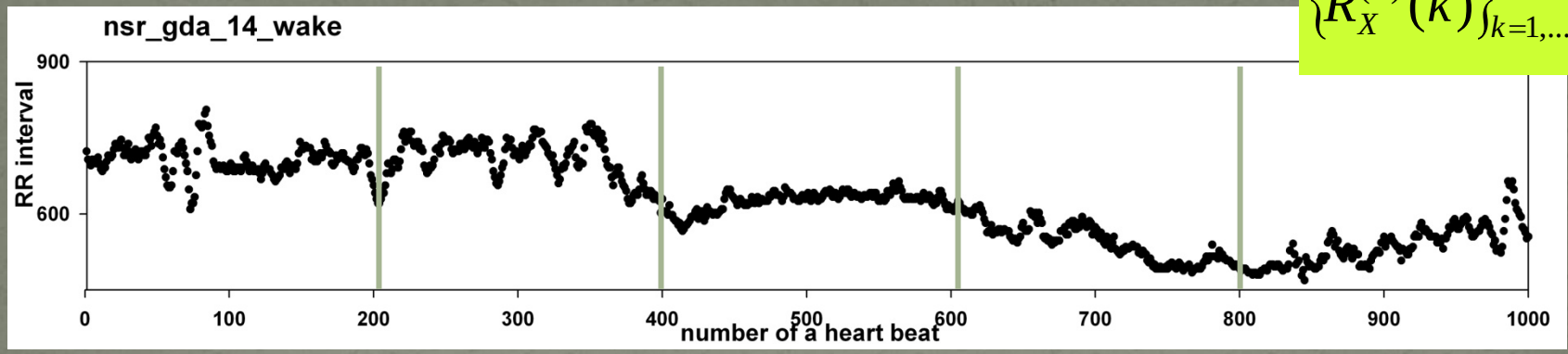
Danuta Makowiec

Instytut Fizyki Teoretycznej i Astrofizyki

UNIWERSYTET GDAŃSKI

Uniwersytet Gdański

Skalowanie:



$$\left\{R_X^{(n)}(k)\right\}_{k=1,\ldots,\frac{N}{n}}$$

Funkcja rozdziału

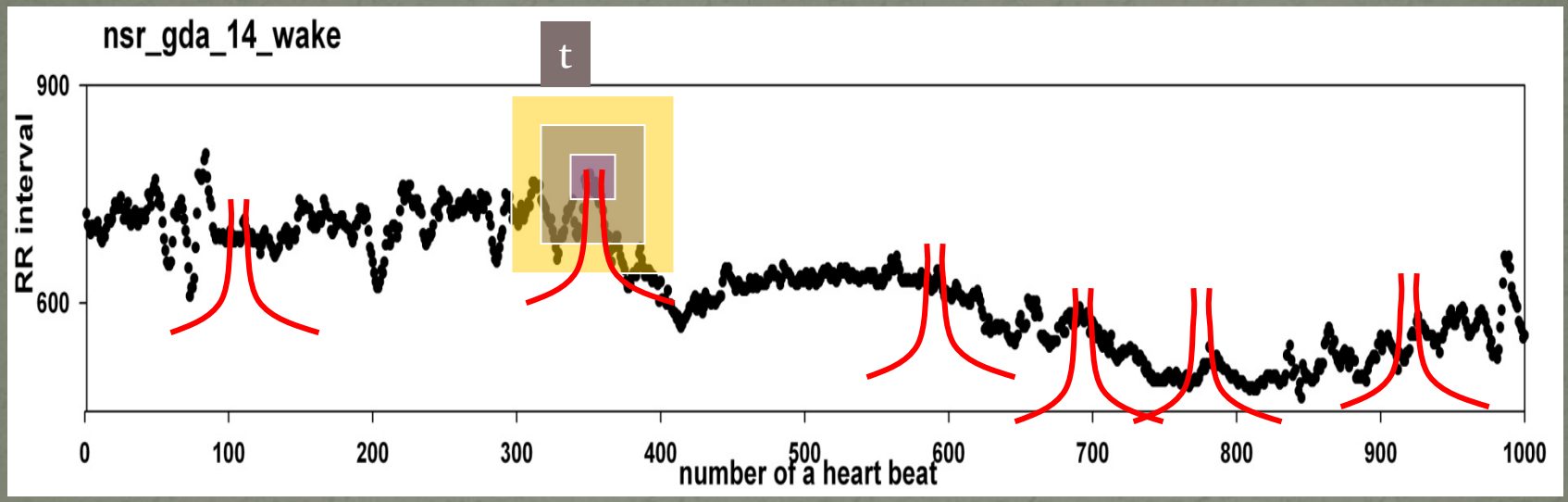
$$\left\langle |R_X^{(n)}(k)|^q \right\rangle_k = ?(n,q)$$

Funkcja wykładnika skalowania

$$\left\langle |R_X^{(n)}(k)|^q \right\rangle_k \propto n^{\tau(q)}$$

Sygnał X(t) ma ***własności skalowania*** jeśli

Widmo multifraktalne



Regularność Holdera w punkcie t:

$$|X(t+\delta)-X(t)| < C|\delta|^{h(t)} \qquad \delta \to 0$$

$$\mathrm{E(h)} = \{t : h(t) = h\}$$

Miara Hausdorffa zbioru o tej samej regularności Holdera

$$h \longrightarrow D(h) = \dim_H \{t : h(t) = h\}$$

Formalizm multifraktalny

Skalowanie:

$$\left\langle |R_X^{(n)}(k)|^q \right\rangle_k \propto n^{\tau(q)}$$

Widmo multifraktalne:

$$h \longrightarrow D(h) = \dim_H \{t : h(t) = h\}$$

$(q, \tau(q))$ Transformacja Legendre $(h, D(h))$

$$h = \frac{d\tau(q)}{dq}$$

$$D(h) = qh - \tau(q)$$

monofraktal

$$\tau(q) = qh - 1$$

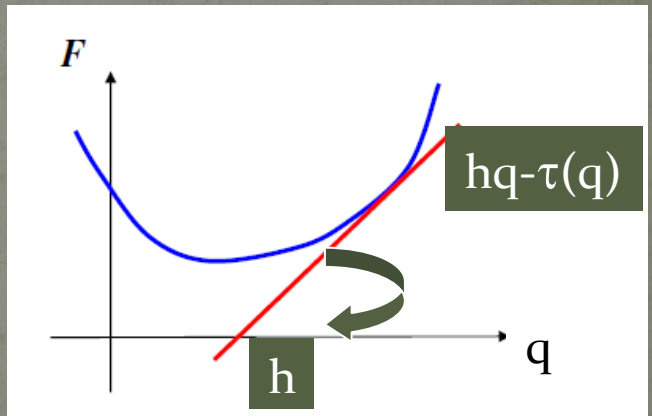
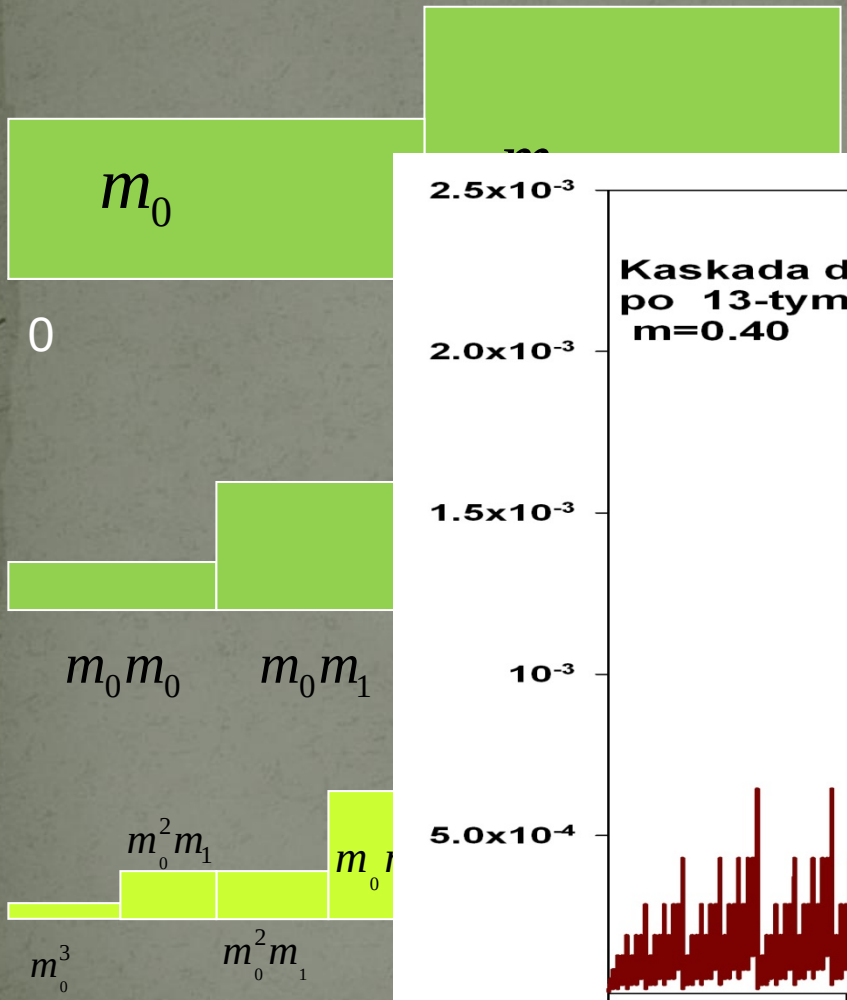
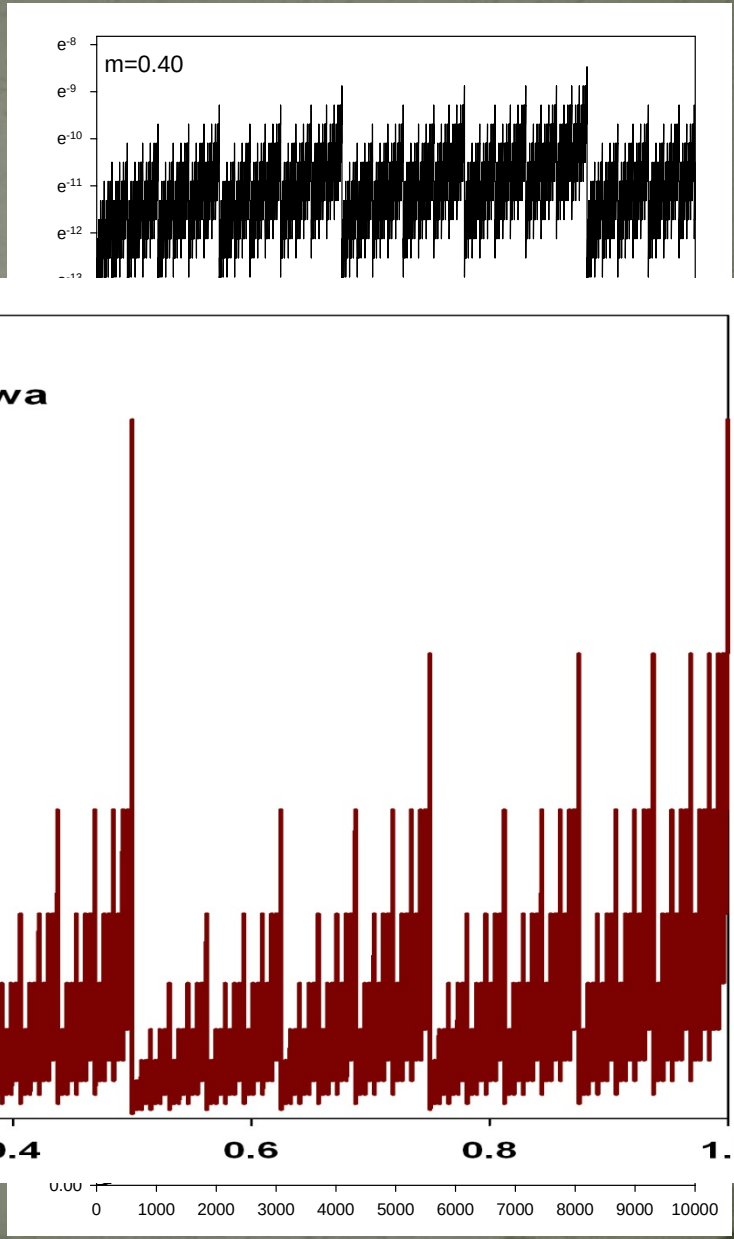
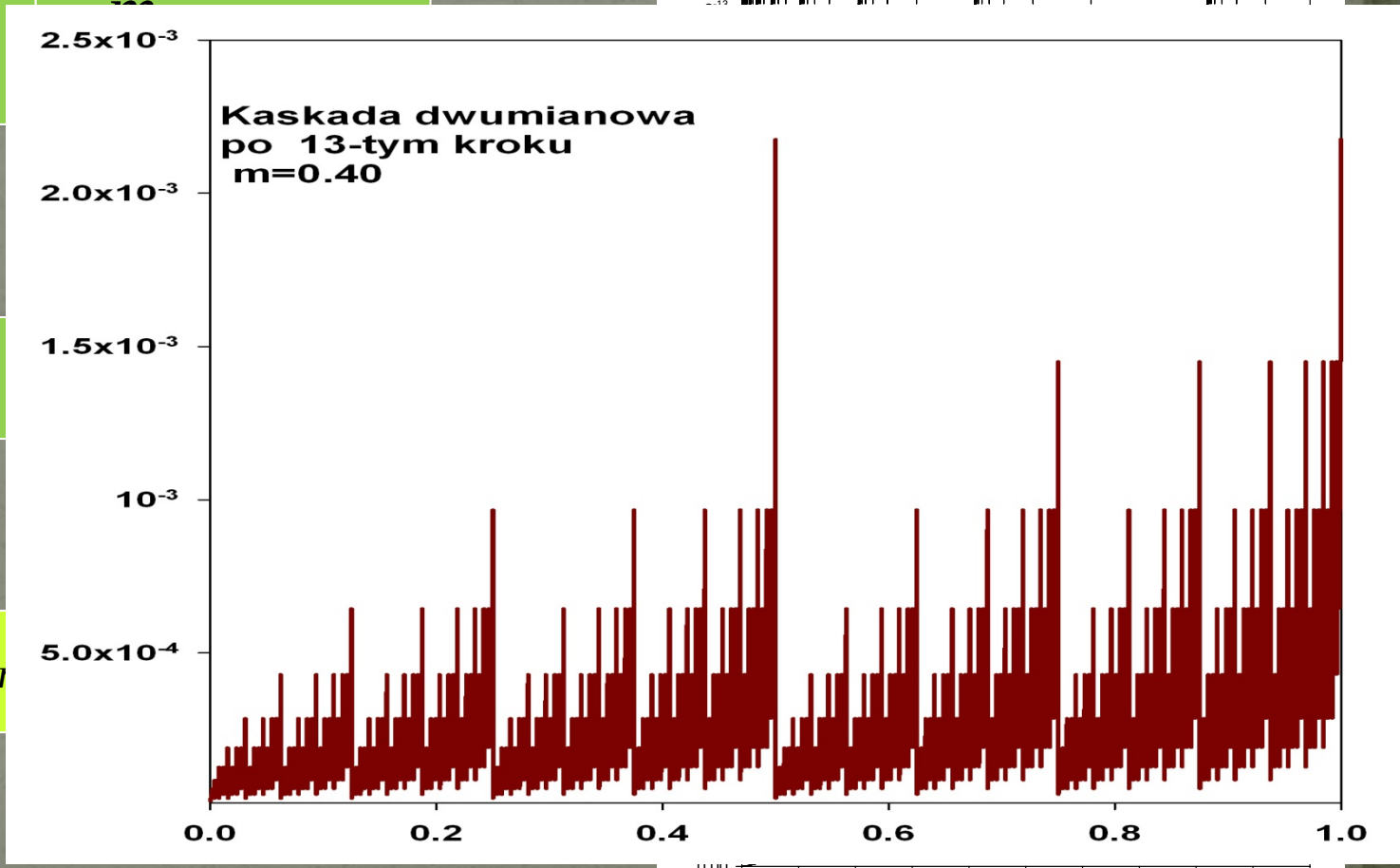


FIG. 1: The graph (blue online) of a convex function $F(x)$. The tangent line at one point is illustrated (red online).

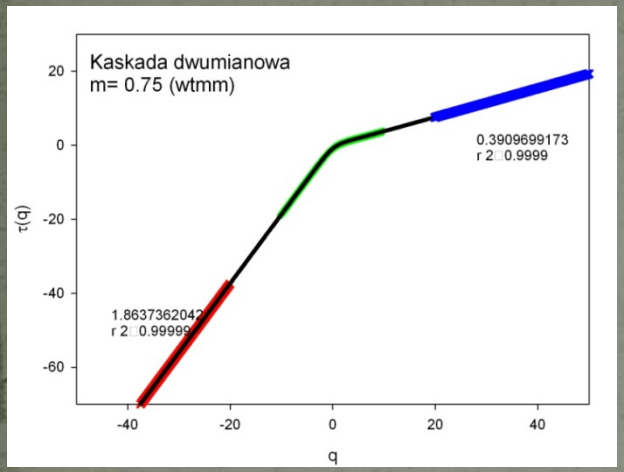
Kaskada dwumianowa:

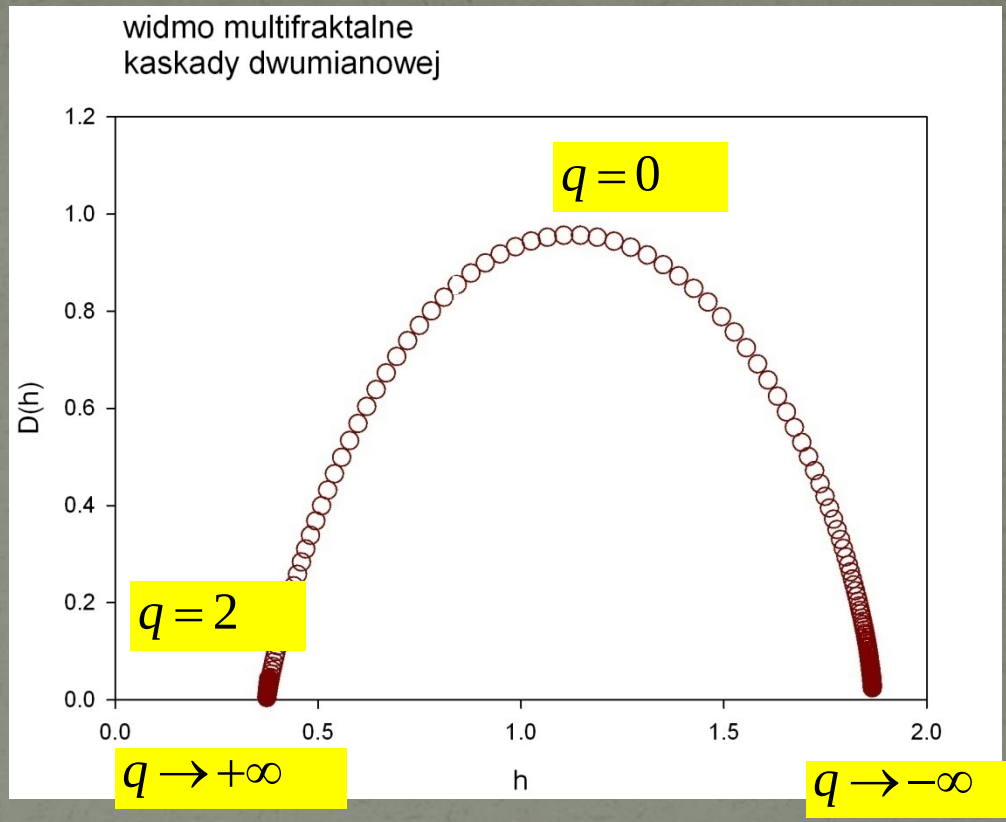






Widmo kaskady





Tradycyjna ocena
widma multifraktalnego

- położenie maksimum
- wykładnik Hursta : korelacje długozasięgowe
- szerokość widma
- „extrema”

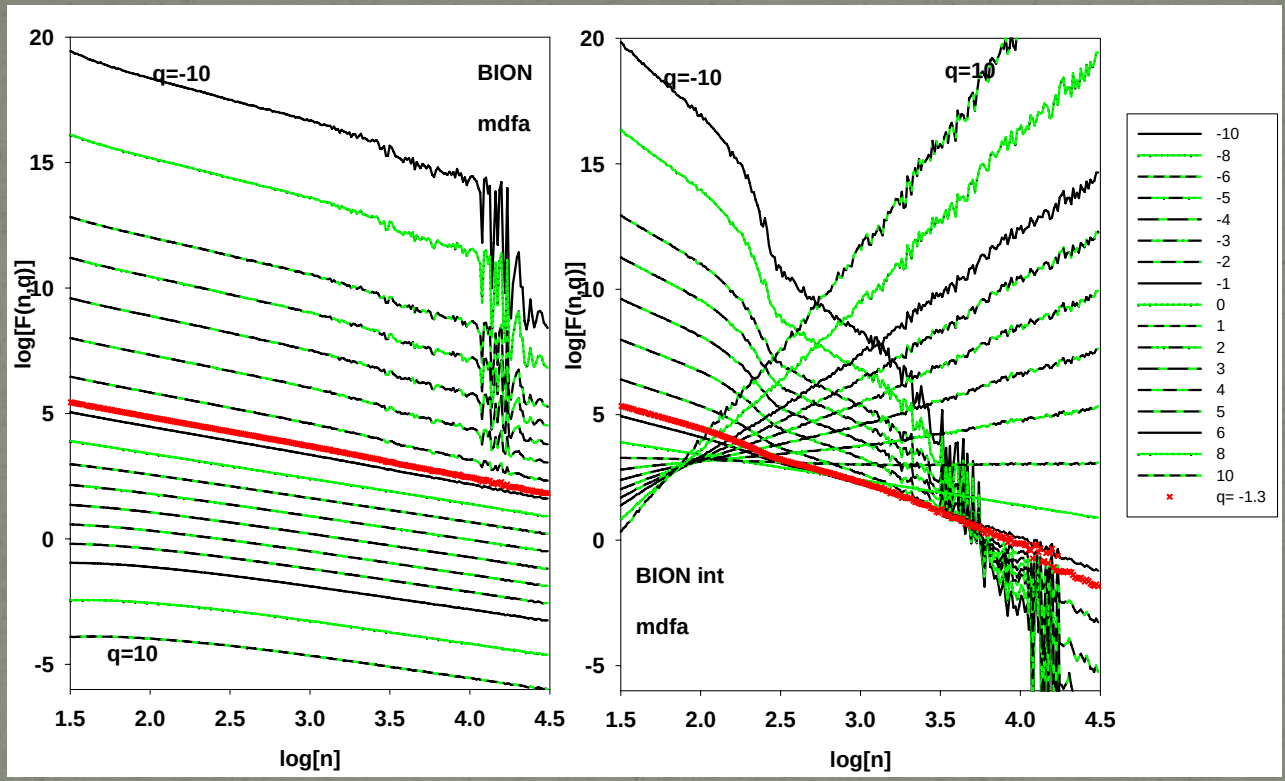
ZATEM:

Jeśli badane dane mają własności skalujące,

to aby te własności sklasyfikować i pomierzyć,
powinno stosować się ANALIZĘ MULTIFRAKTALNĄ

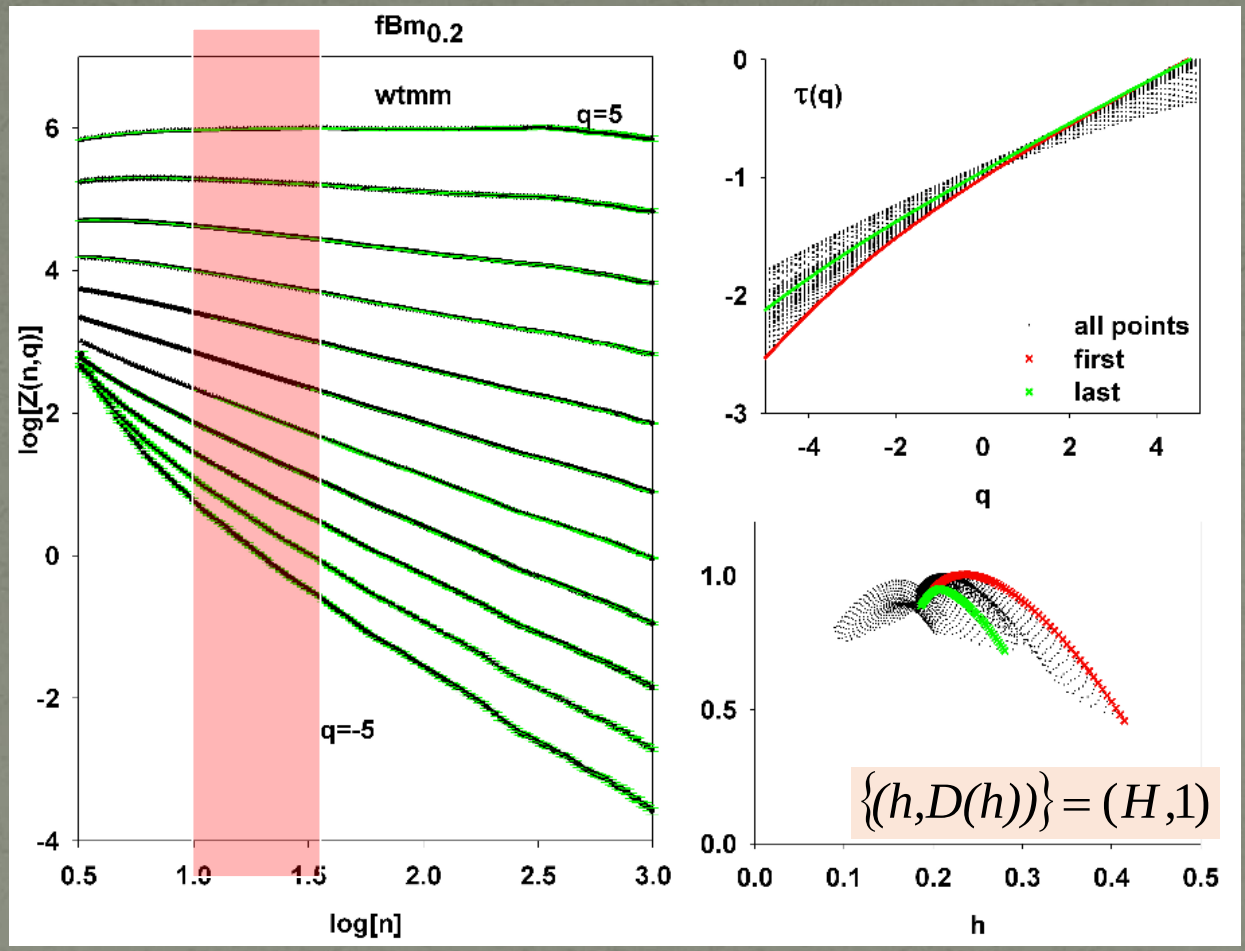
Zyskujemy zwartą, uproszczoną reprezentację danych

Dane rzeczywiste :



Artefakty czy informacja o zjawisku?

Formalizm multifraktalny w praktyce: WTMM



$$Z(n,q) \propto n^{\tau(q)}$$

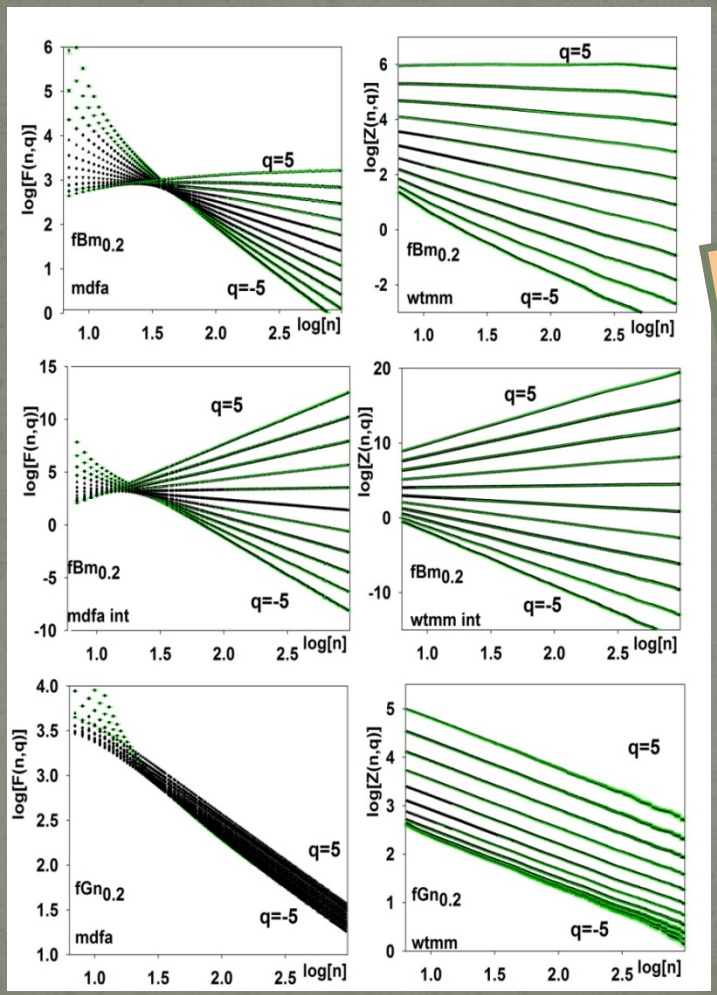
fBm_0.2

MDFA WTMM

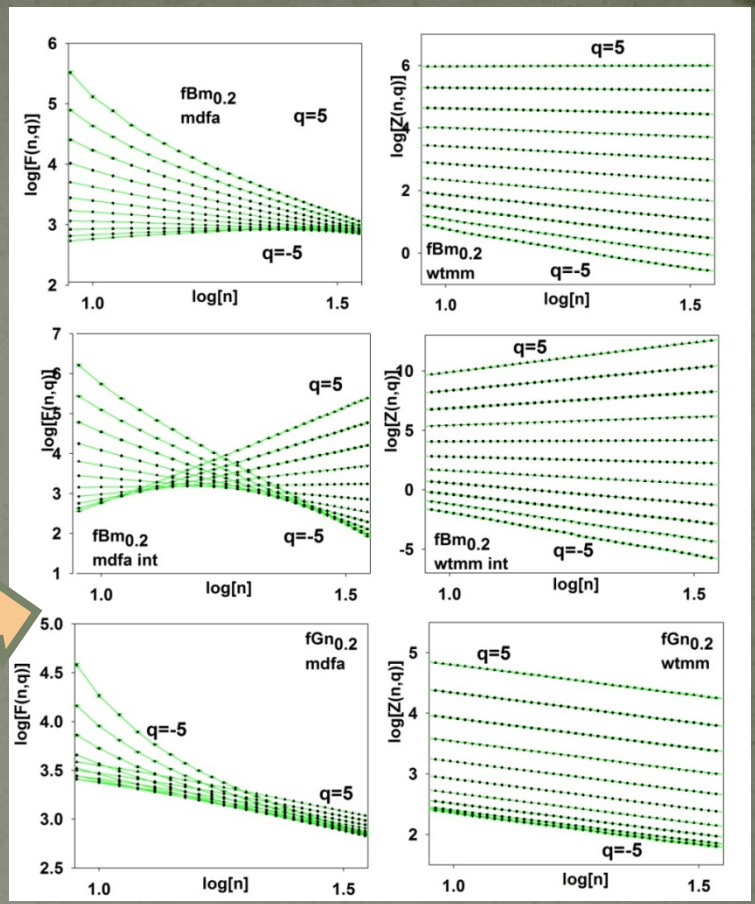
seria

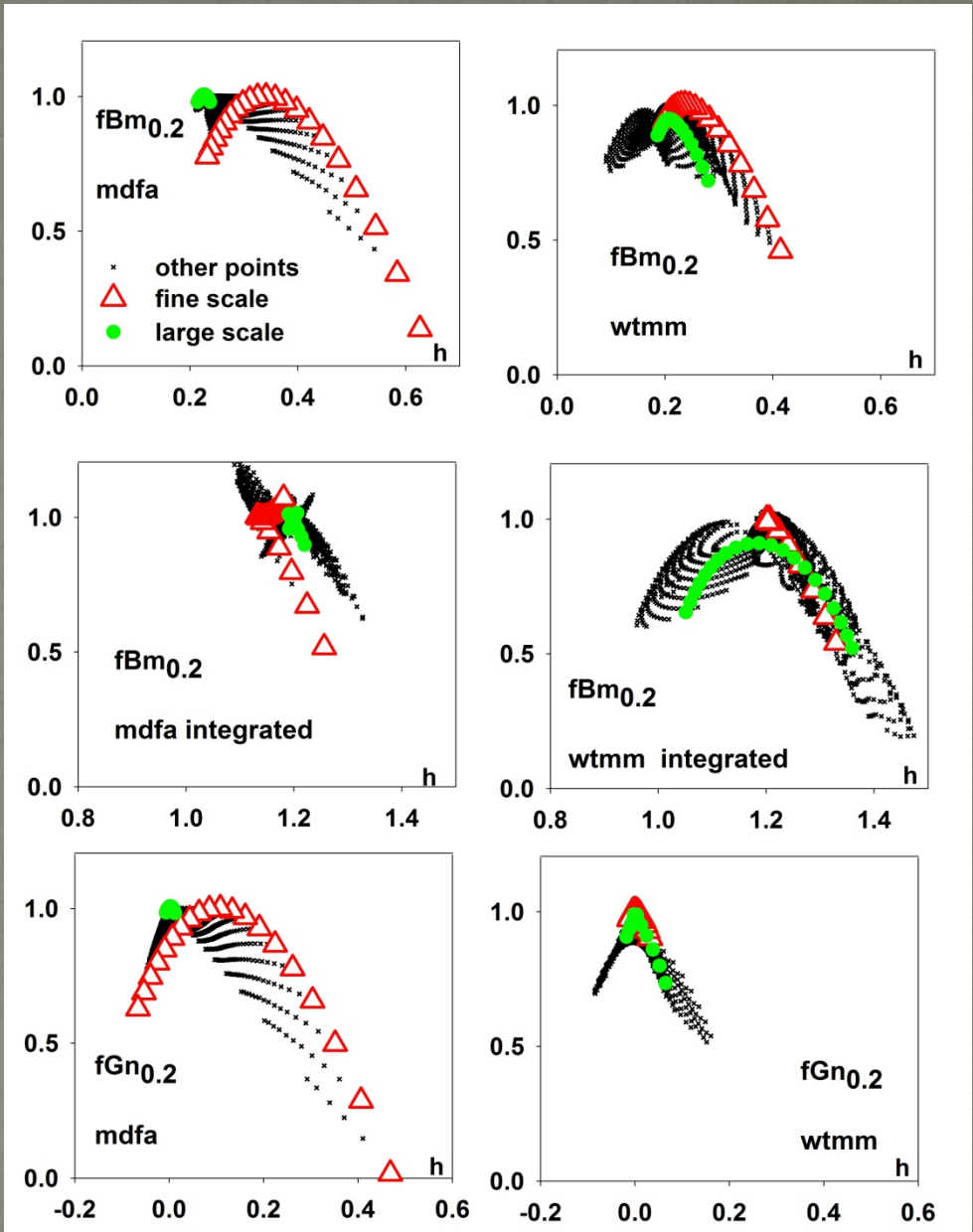
seria wysu-mowa-na

seria zróżni-cowana



zoom



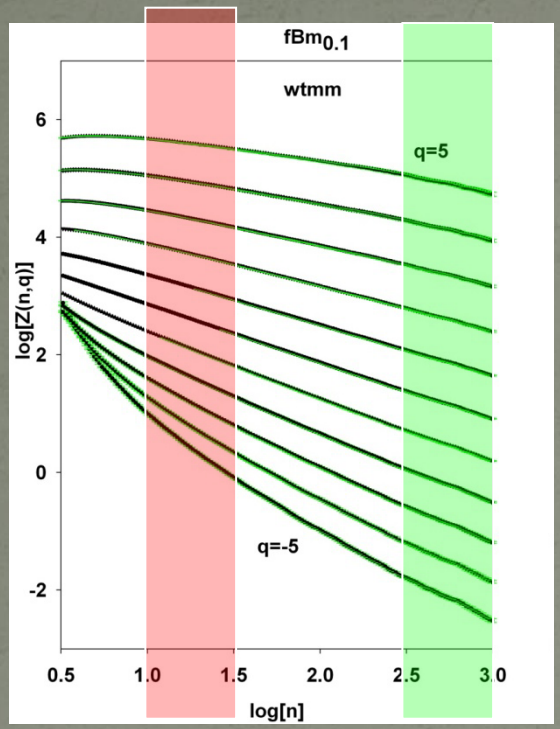


$\{(h,D(h))\} = (H,1)$

$\{(h,D(h))\} = (1+H,1)$

$\{(h,D(h))\} = (0,1)$

Skalowanie w sercowych przedziałach skal



DLACZEGO?

1.0…..1.5

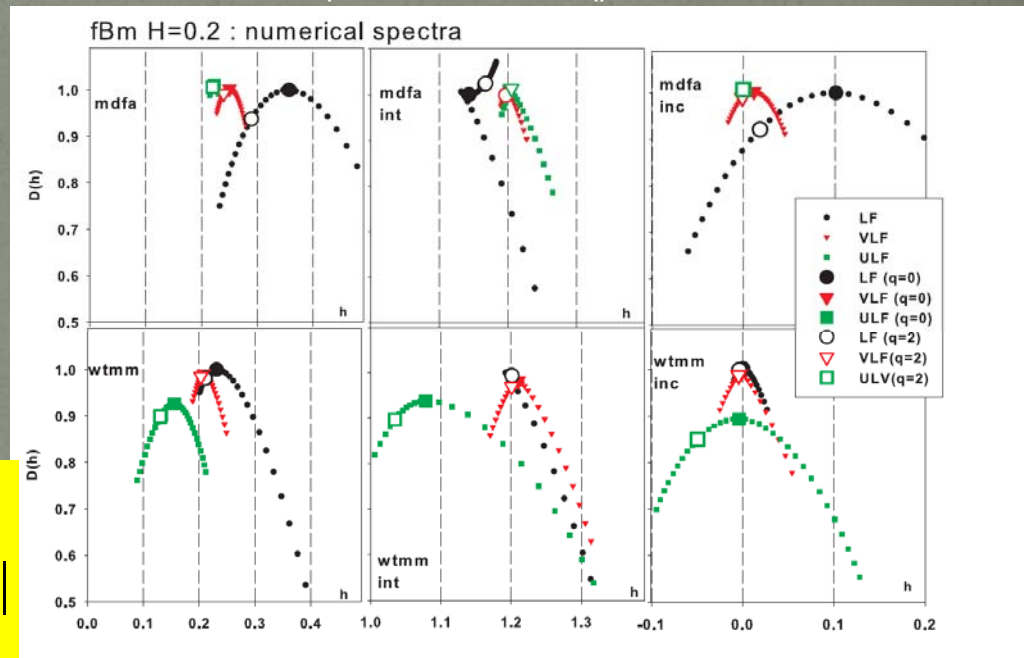
odpowiada n: 10 do 30 (8s…25s) : **LF**

1.5…..2.5

odpowiada n 30 to 300 (8s.. do 4 min): **VLF**

2.5…..3.0

odpowiada n > 300 (ponad 4 min : **ULF**



$$\mathrm{MAX}: h(q=0)$$

$$1/2\ \text{width}: |h(q=0)-h(q=2)|$$

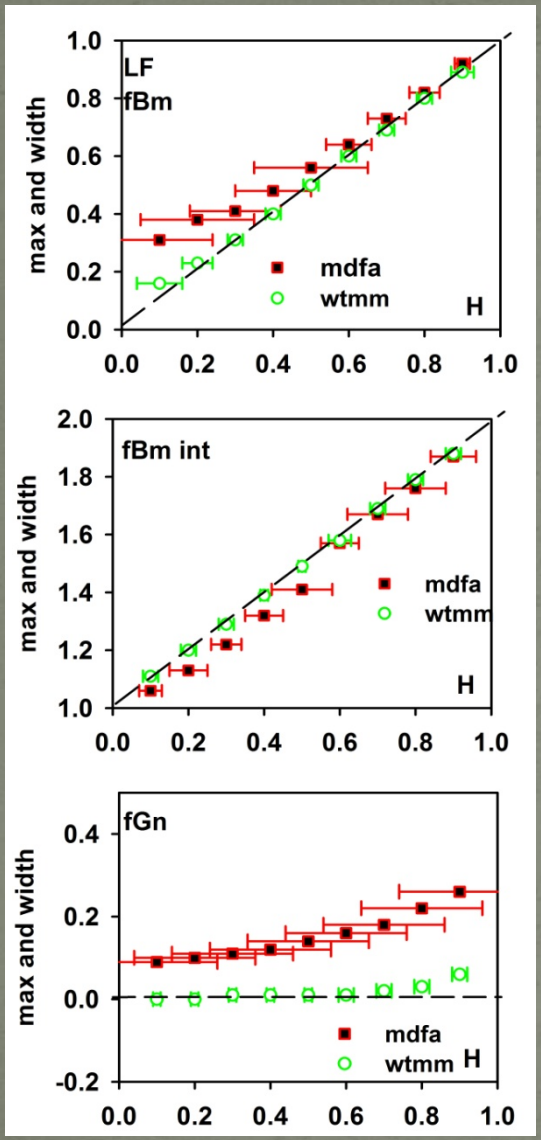
$$\text{extreme}: h(q=5)$$

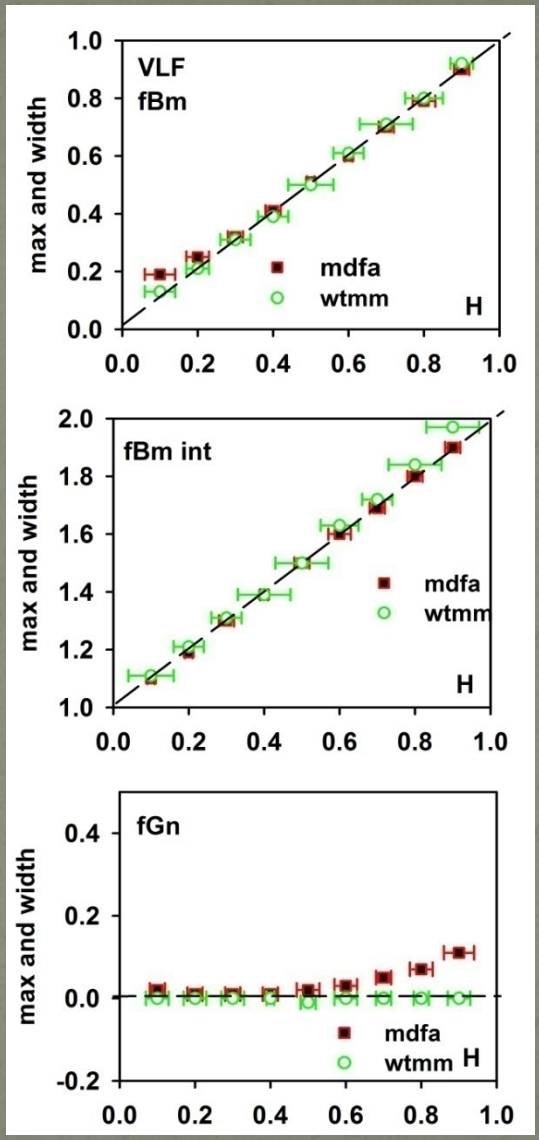
$$\{(h,D(h))\}=(H,1)$$

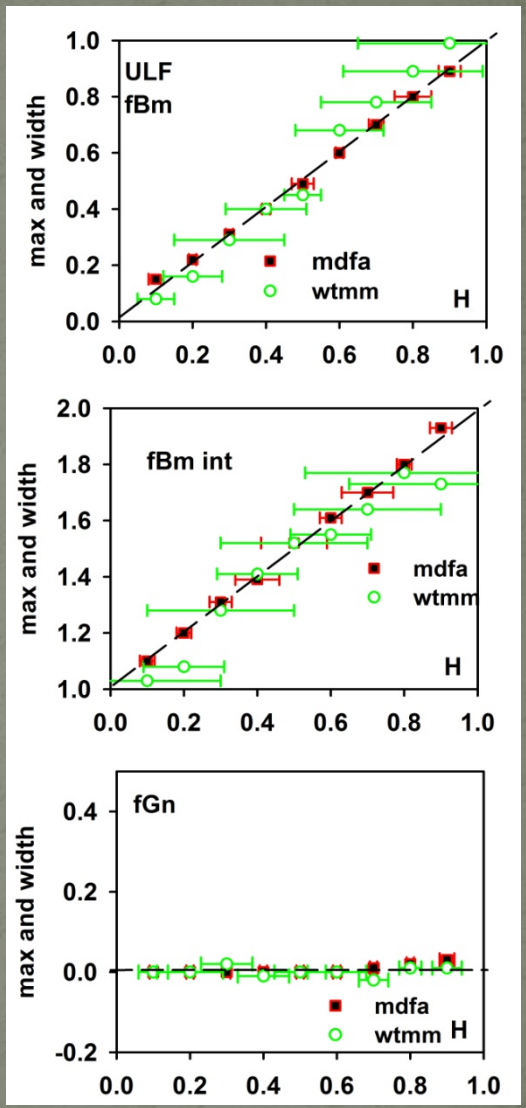
$$\{(h,D(h))\}=(1+H,1)$$

$$\{(h,D(h))\}=(0,1)$$

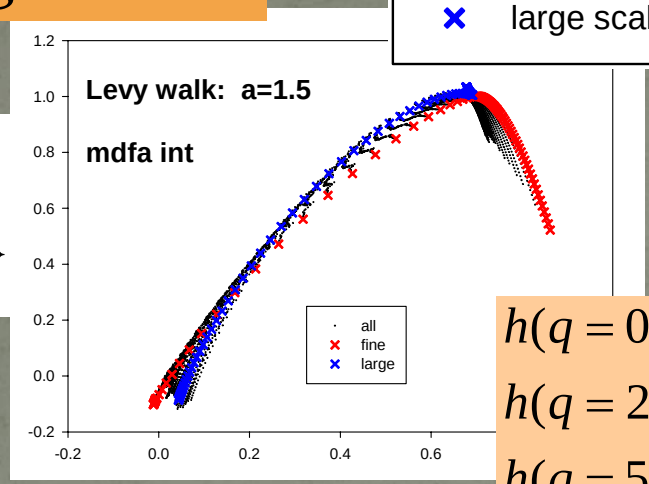
Rezultaty dla fBm_H:





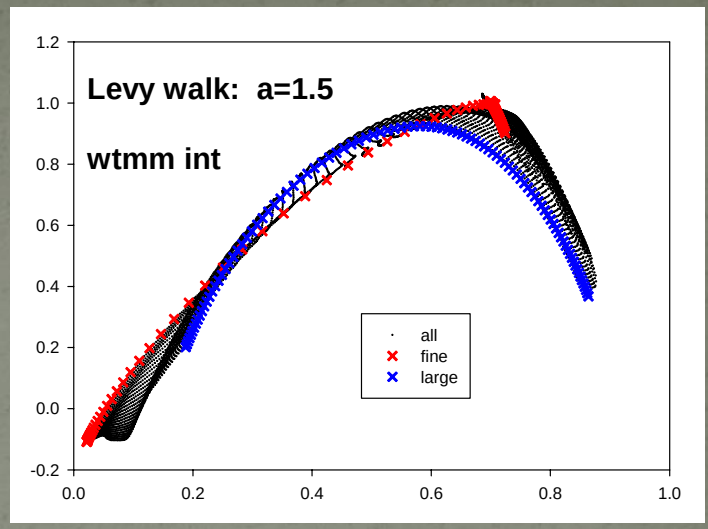


Błądzenie i szum Levy'ego:



$\{(h,D(h))\} =$

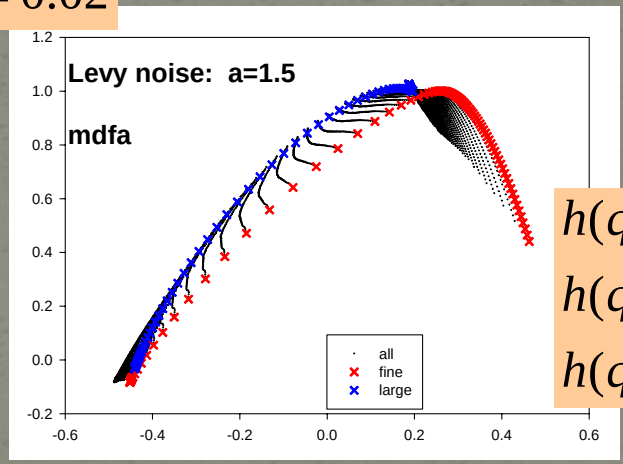
$\{(0,0),(\frac{1}{\alpha},1)\}$



$h(q=0)=0.68$

$h(q=2)=0.25$

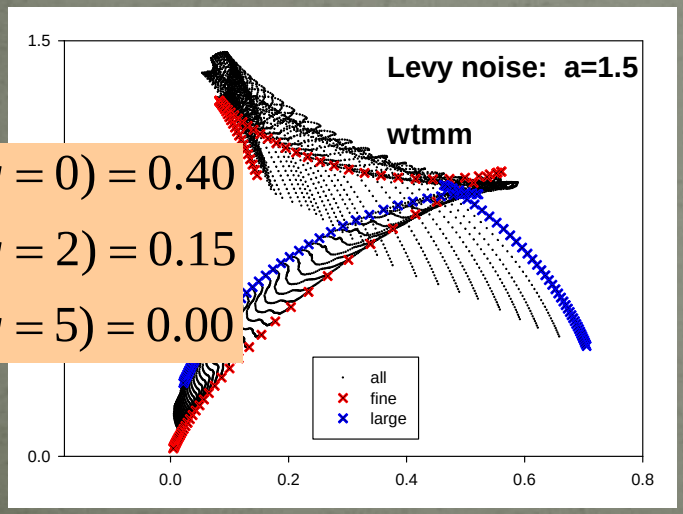
$h(q=5)=0.02$



$h(q=0)=0.18$

$h(q=2)=-0.25$

$h(q=5)=-0.48$



$h(q=0)=0.40$

$h(q=2)=0.15$

$h(q=5)=0.00$

$\{(h,D(h))\} =$

$\{(-0.5,0),(\frac{1}{\alpha}-0.5,1)\}$

Kaskada dwumianowa: m=0.75

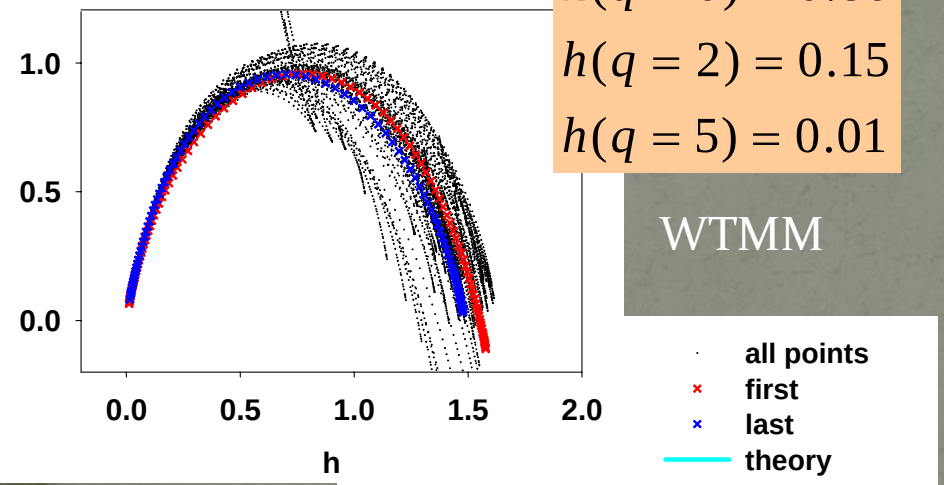
noise

$h(q=0)=0.86$

$h(q=2)=0.15$

$h(q=5)=0.01$

WTMM



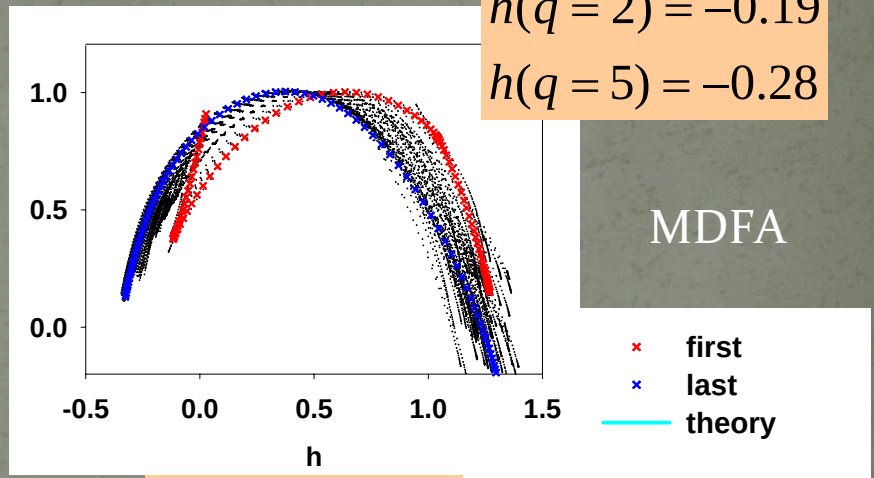
noise

$h(q=0)=0.44$

$h(q=2)=-0.19$

$h(q=5)=-0.28$

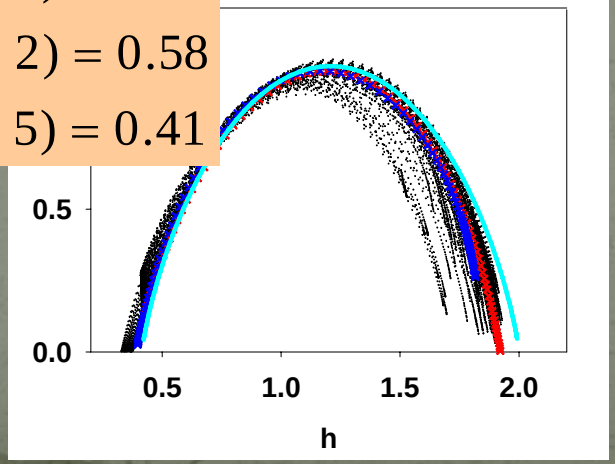
MDFA



$h(q=0)=1.23$

$h(q=2)=0.58$

$h(q=5)=0.41$

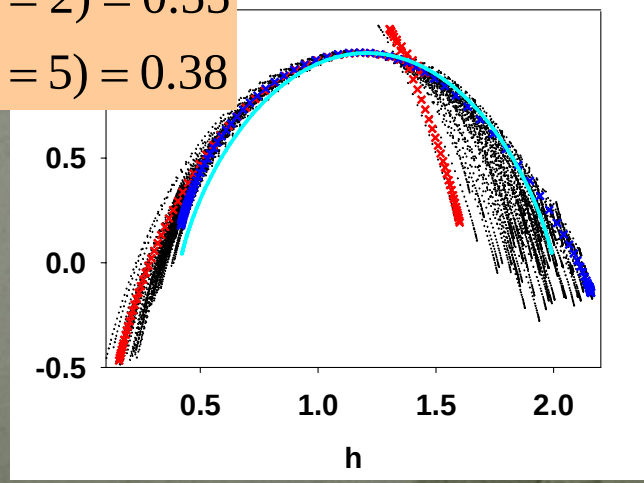


walk

$h(q=0)=1.23$

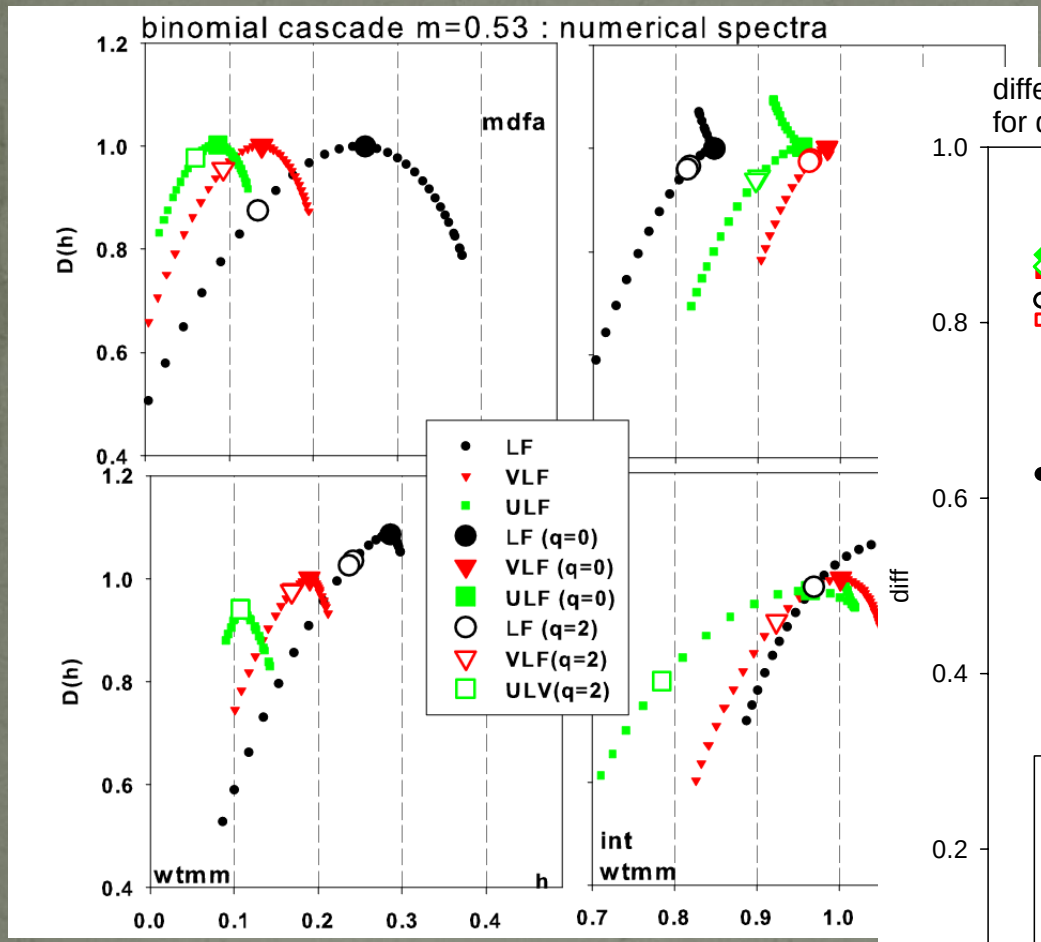
$h(q=2)=0.55$

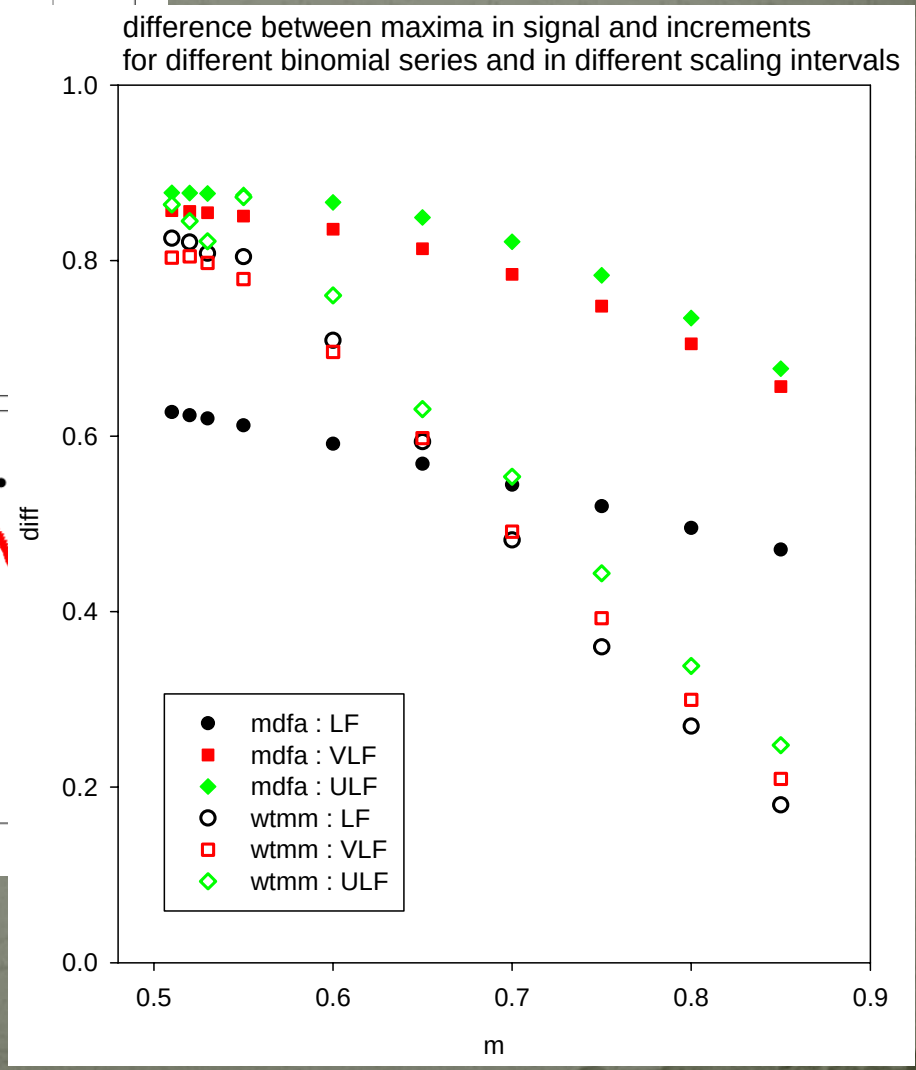
$h(q=5)=0.38$



walk

Rezultaty dla kaskady m=0.53 w „sercowych” przedziałach

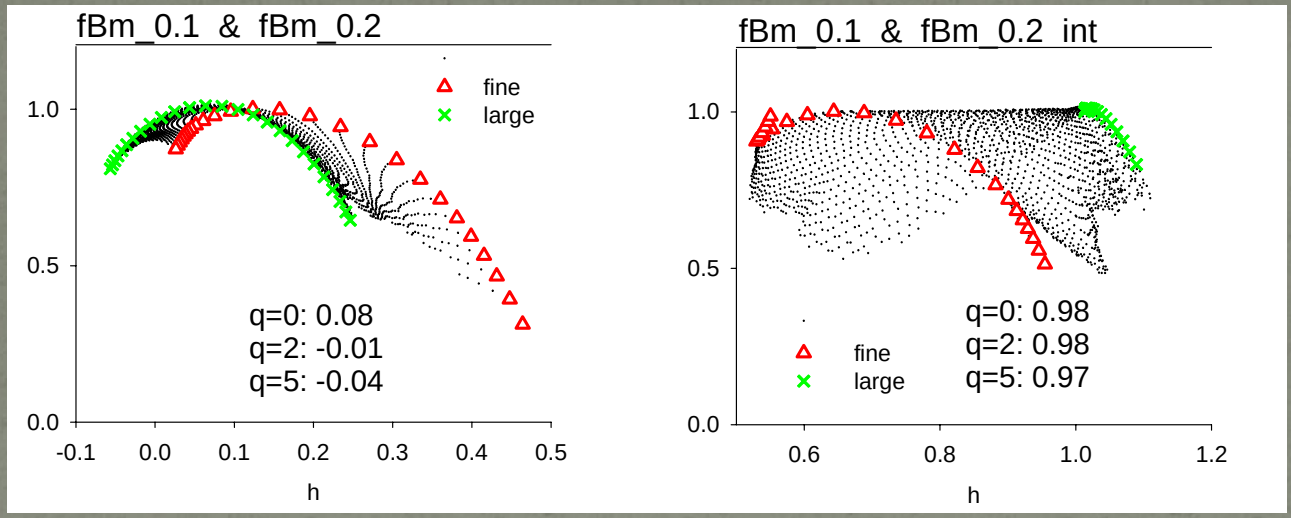


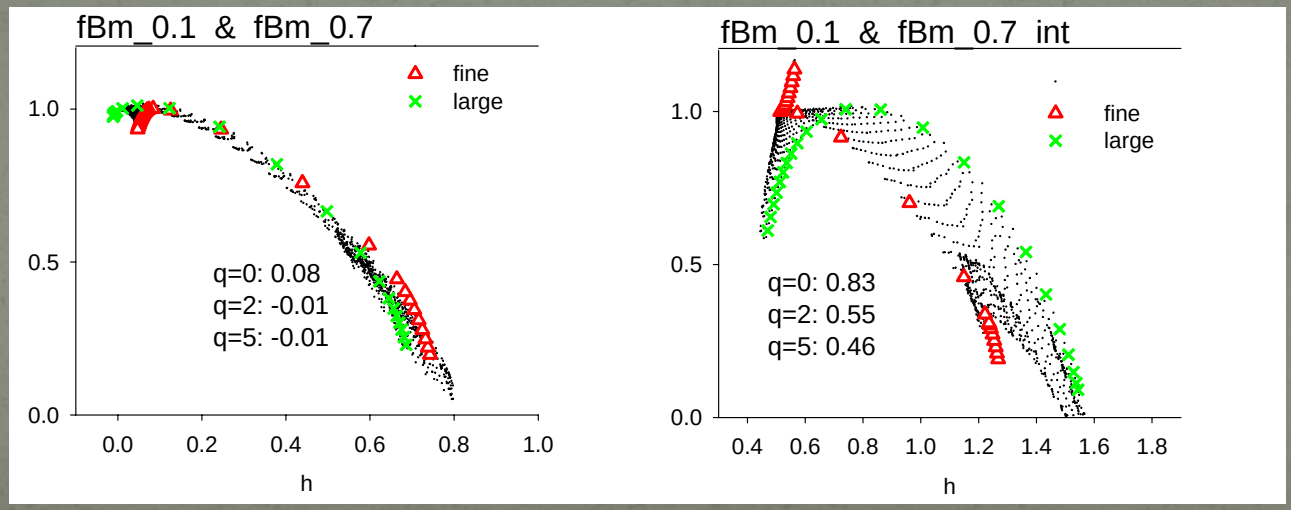


Modyfikacje prostych sygnałów:

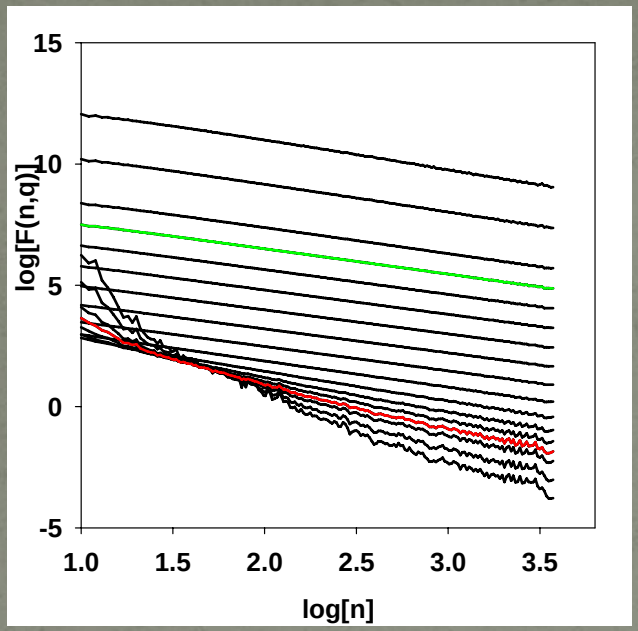
- Przełączanie losowe pomiędzy dwoma procesami monofraktalnymi
- Przełączanie systematyczne pomiędzy dwoma procesami monofraktalnymi
- Wydłużenie kroku kaskady dwumianowej

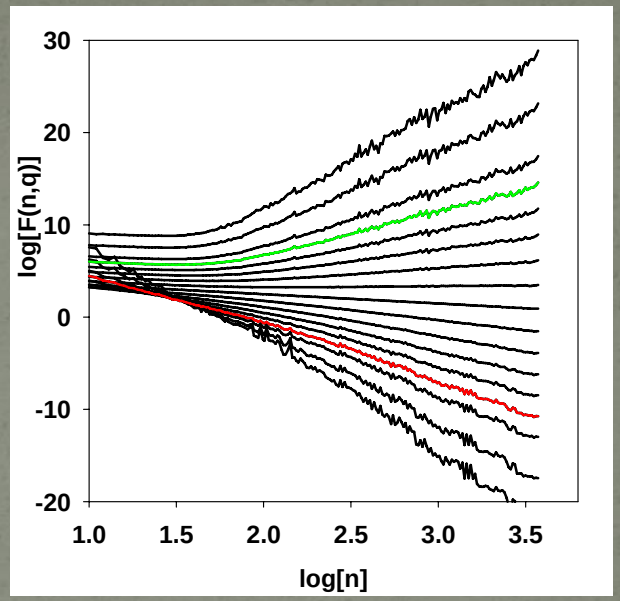
Losowe przełączanie między dwoma fBm_H

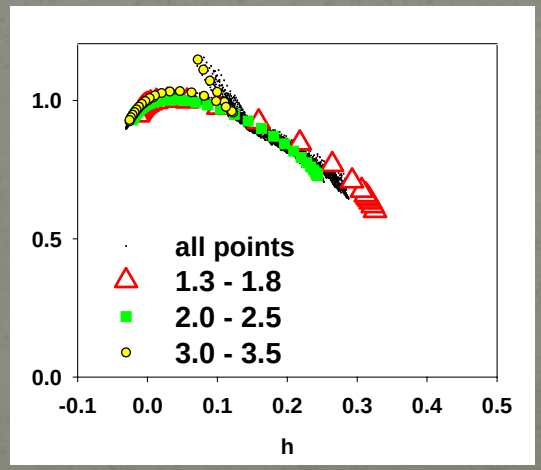


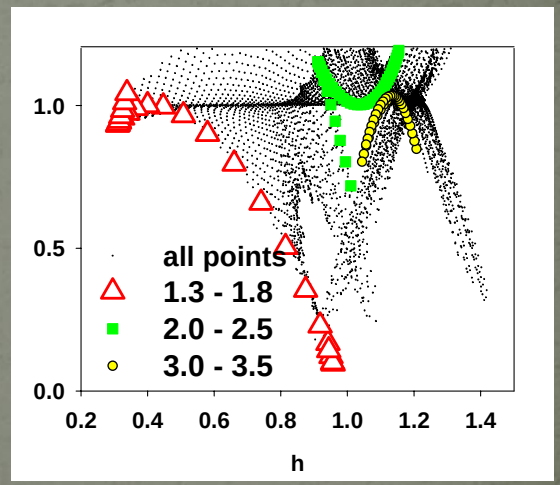


Stałe przełączanie między dwoma fBm_H co krok:

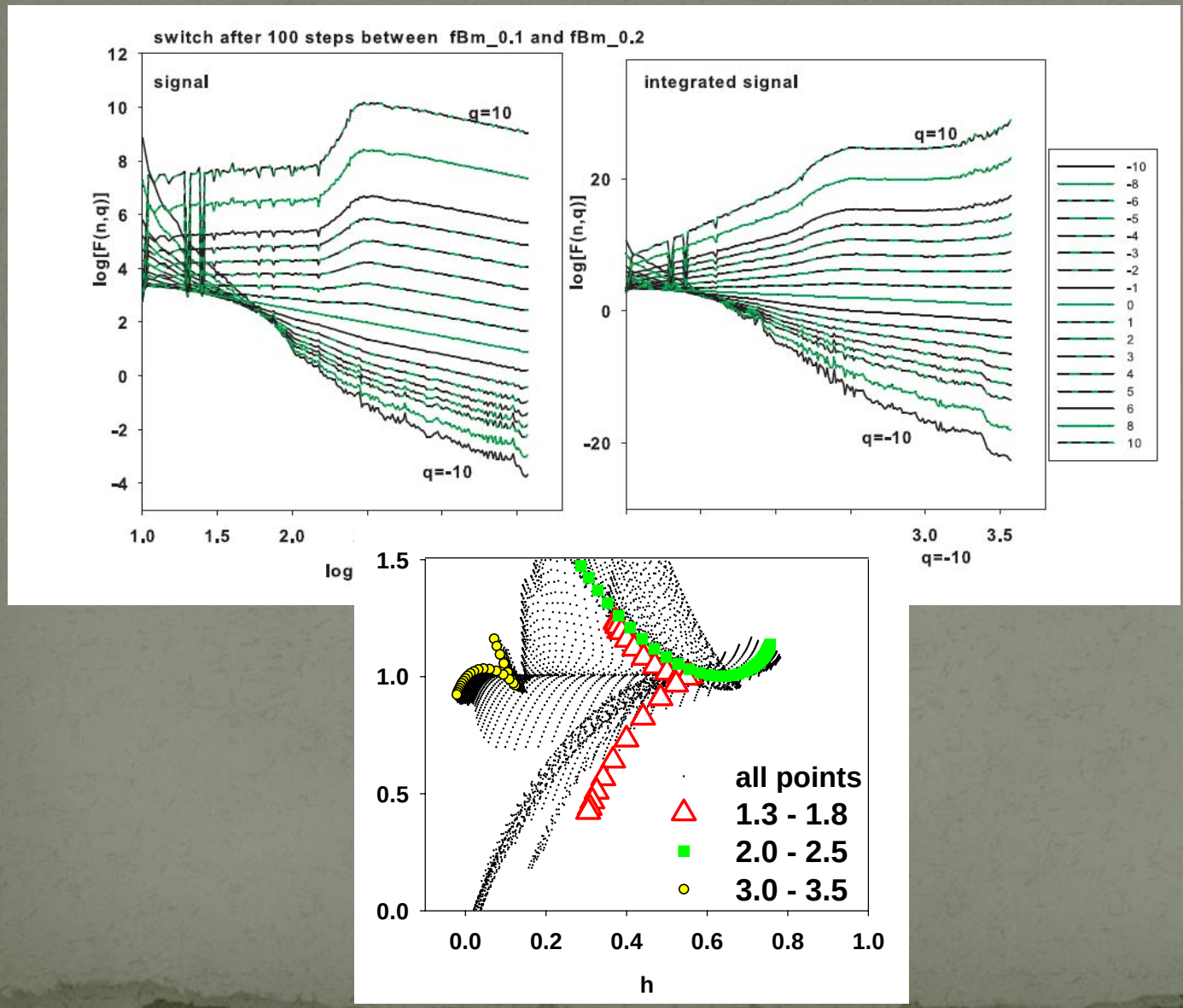






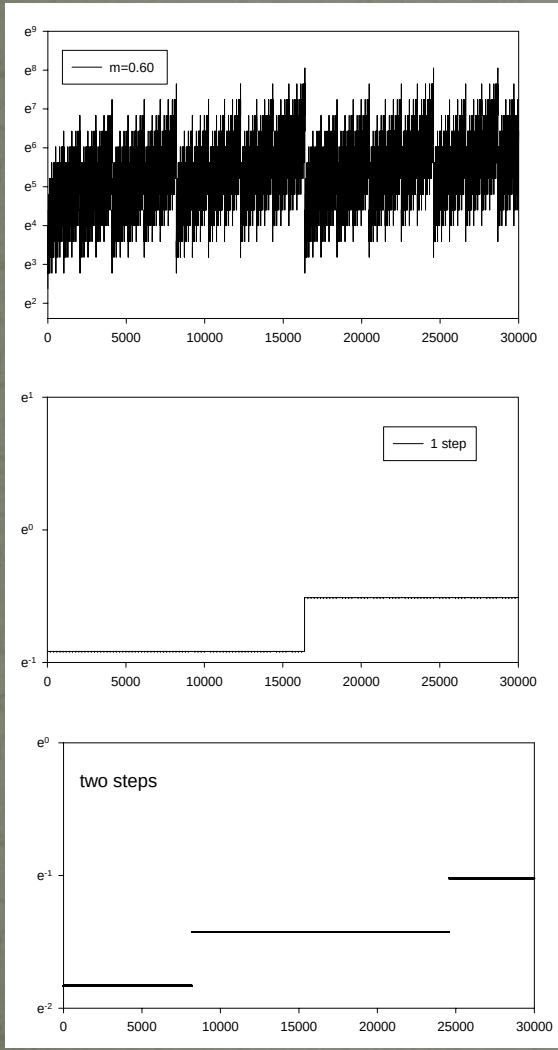


Stałe przełączanie między dwoma fBm_H co 100 kroków



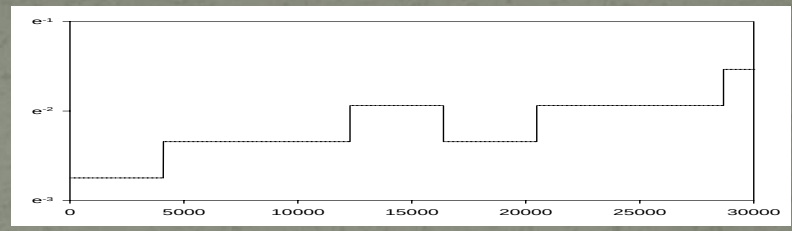
A jeśli kaskada dwumianowa dokonuje się wolniej.....

Połowienie co 2^16 kroków

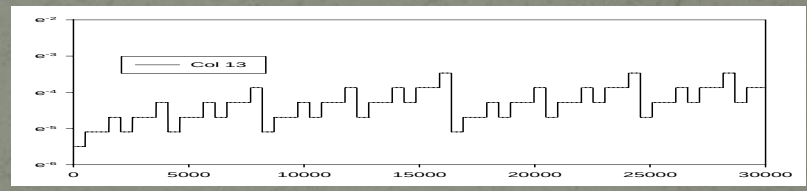


Połowienie co 2^15 kroków

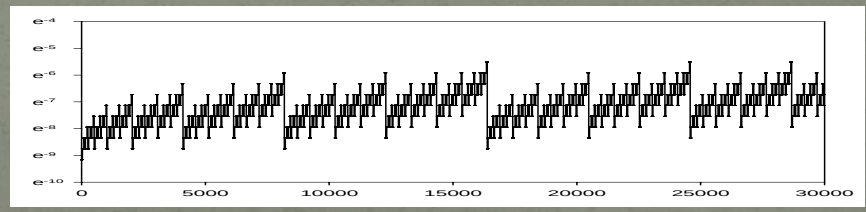
Połowienie co 2^14 kroków



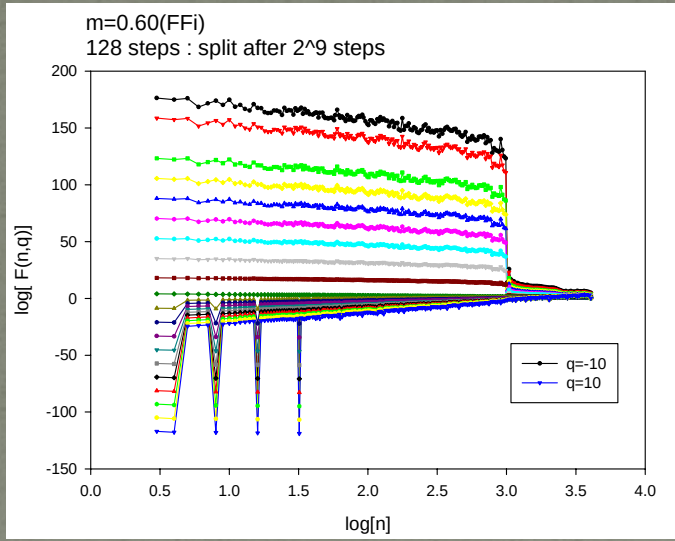
Połowienie co 2^8 kroków

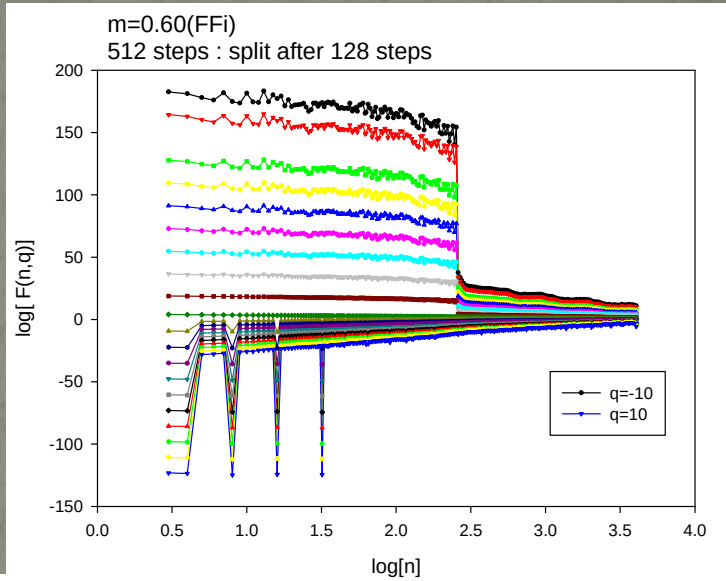


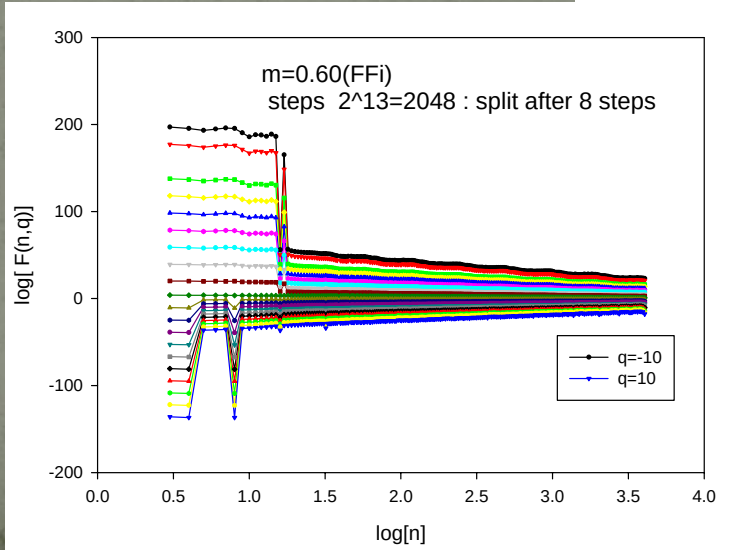
Połowienie co 2^5 kroków



Funkcje rozdziału:







Wyznaczanie własności multifraktalnych w sygnałach rzeczywistyczych

Własności skalujące powinny być wyznaczane dla różnych funkcji rozdziału.

Własności skalujące powinny być wyznaczane przy ustalonym przedziale skalowania.

Analiza multifraktalna powinna być prowadzona równolegle:

dla sygnału

dla sygnału wysumowanego, jeśli pojawia się widmo z h<1

dla sygnału przyrostów, jeśli pojawia się widmo z h >1

Ocena widm multifraktalnych:

-Wyznaczenie maksimów widm

-Wyznaczenie różnicy pomiędzy maksimami widm

-Oszacowanie szerokości widma

Praktycznie:

h (q=0)

szerokość

Odległość maximów

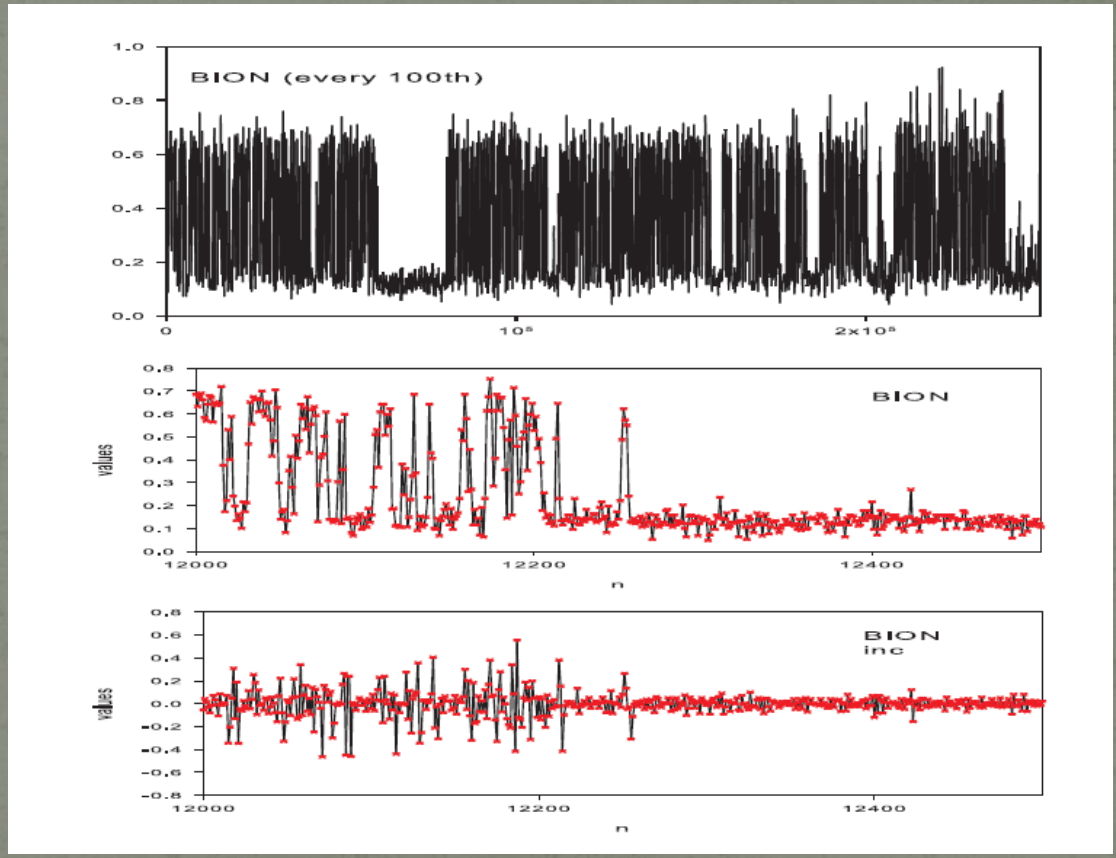
h(q=2)

Hurst

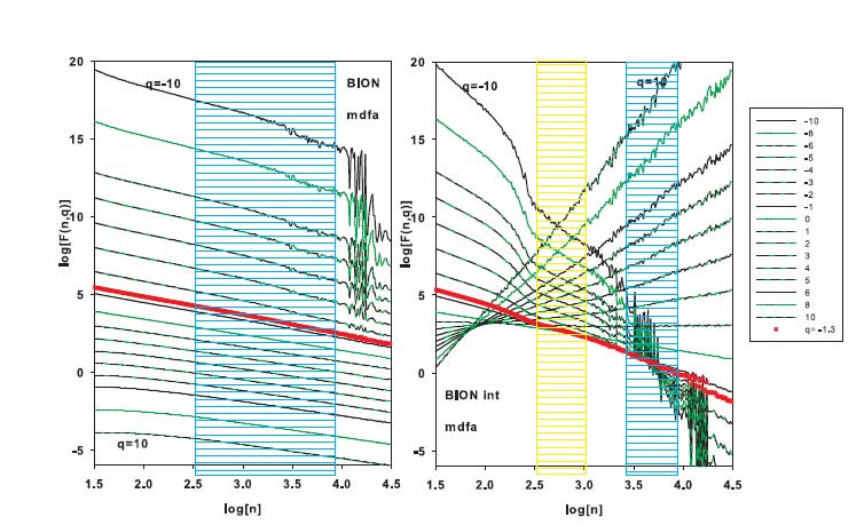
h(q=5)

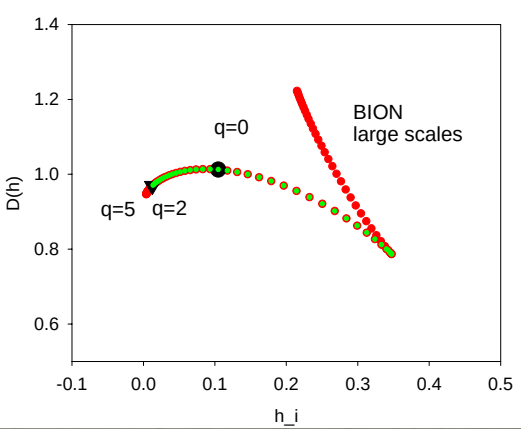
extrema

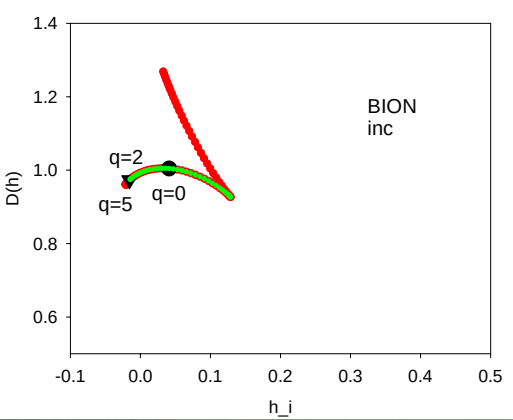
Prąd przepływający przez kanał potasowy:

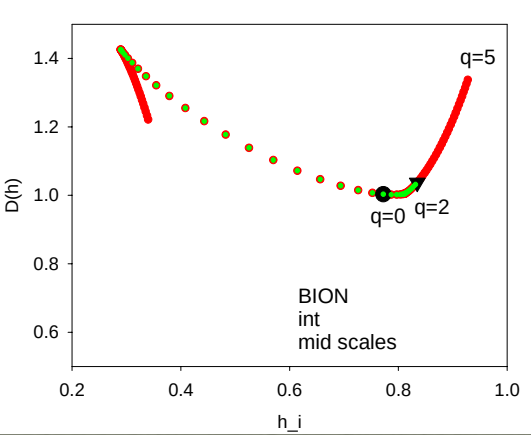


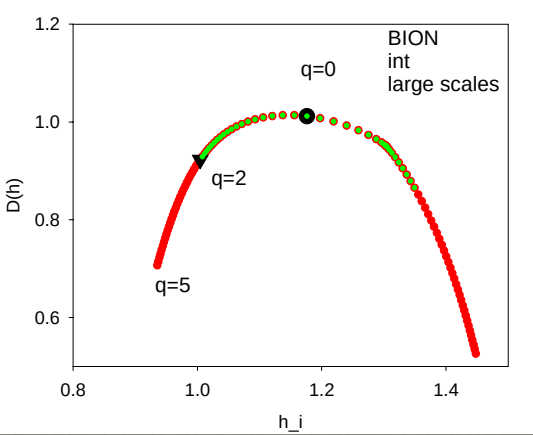
Analiza multifraktalna dla BION :











Dziękuję za uwagę