The Lehman Brothers Effect and Bankruptcy Cascades

Pawel Sieczka(1), Didier Sornette(2) and Janusz A. Holyst(1)

(1) Faculty of Physics, Warsaw University of Technology, Poland
(2) ETH Zurich and Swiss Finance Insititute, Switzerland

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Variable R evolves in time according to:

$$R(t) = R(t-1) + s(t-1), \tag{1}$$

where $s(t)$ is a stochastic variable and $s(t) = -1, 0, 1$.

For a set of N firms, we define the conditional probability for a variable s_i, $i = 1, 2, ..., N$:

$$P(s_i \setminus s_1, ..., s_{i-1}, s_{i+1}, ..., s_N, R_i) = \\ = \frac{1}{Z} \exp(\sum_{j \neq i} J_{ij} \delta(s_i, s_j) + f(R_i, s_i)). \quad (2)$$

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$$P(J_{ij}) = \frac{1}{\sqrt{2\pi\sigma_J^2}} \exp(-\frac{(J_{ij} - J_0)^2}{2\sigma_J^2}). \quad (3)$$

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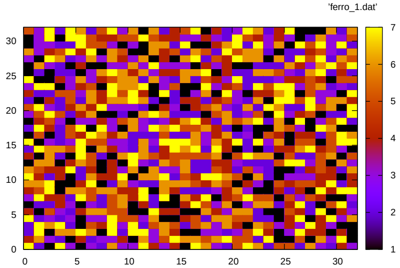
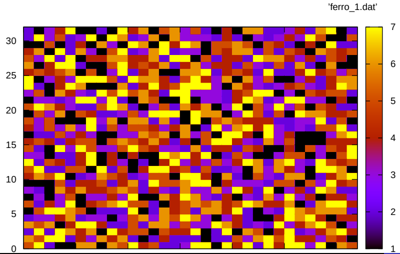
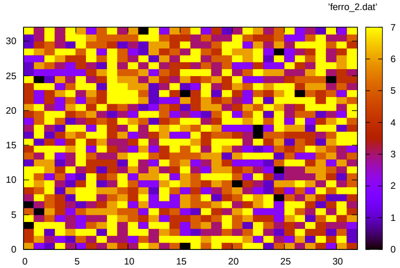
- asynchronous dynamics

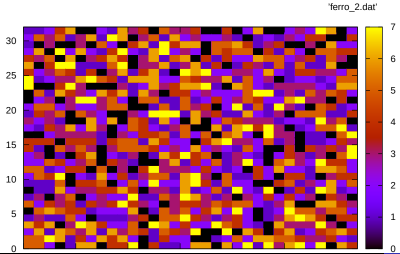




Evolution of a portfolio in a ferromagnetic phase: $J_0 = 0.02$, $\sigma = 0.001$.

Rating scale:
$R = 0, \ldots \; R = 7$





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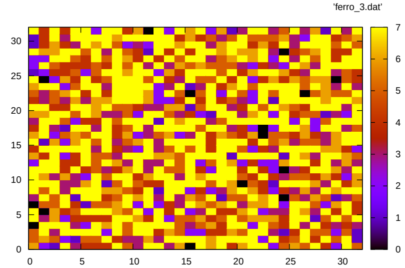

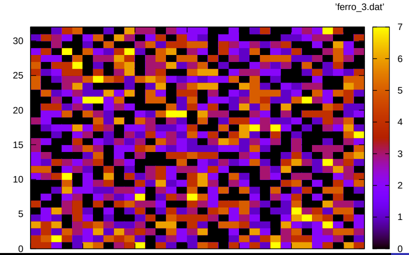

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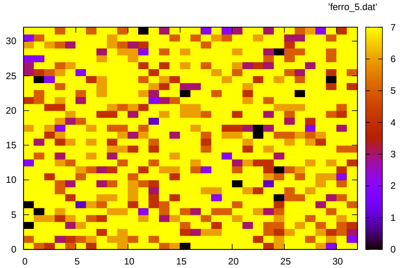

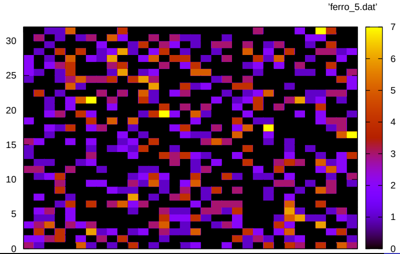

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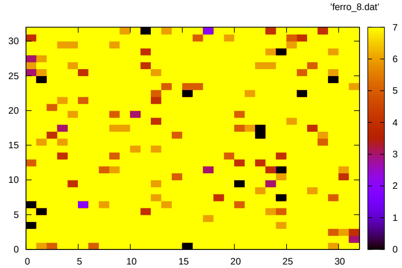

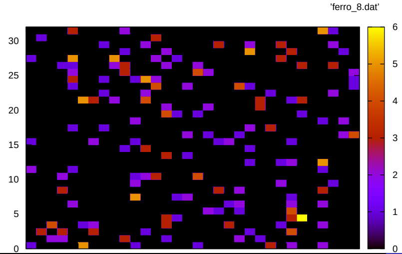

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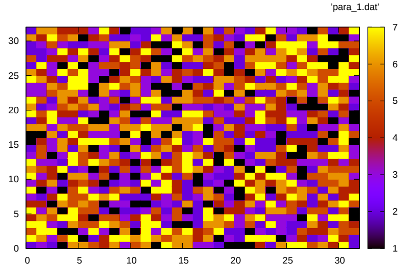
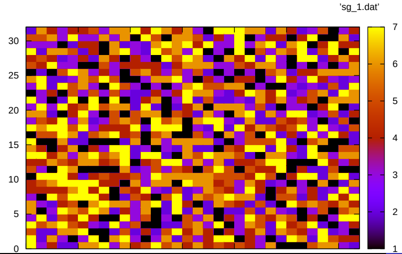
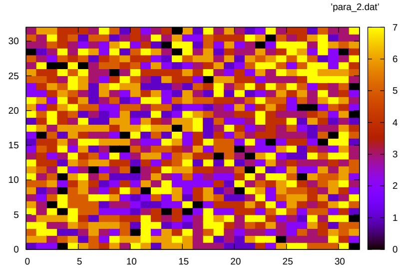
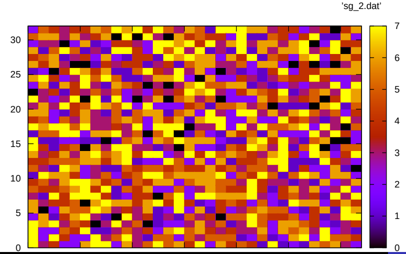




Evolution of a portfolio in a paramagnetic phase: $J_0 = 0.0001$, $\sigma = 0.001$ (top) and a spin-glass phase: $J_0 = 0.0001$, $\sigma = 0.1$ (bottom) .

Rating scale:
$R = 0, \dots \; R = 7$





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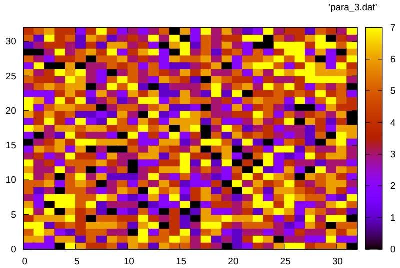

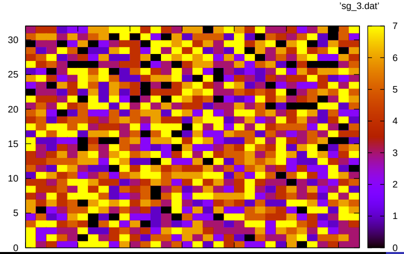

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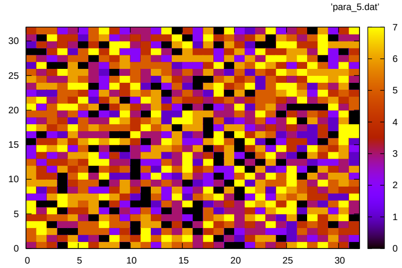

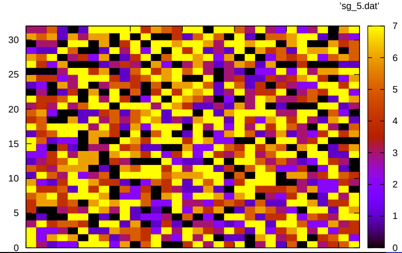

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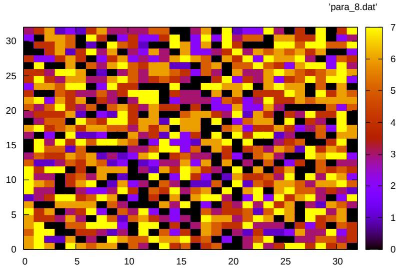

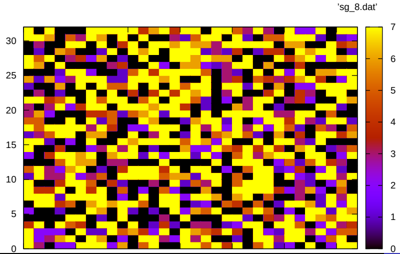

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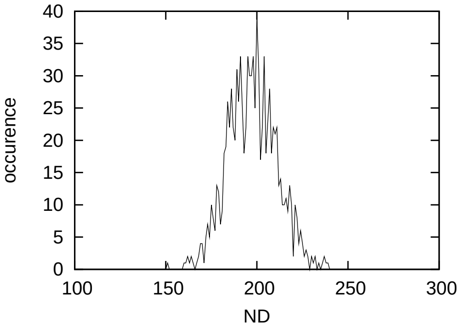
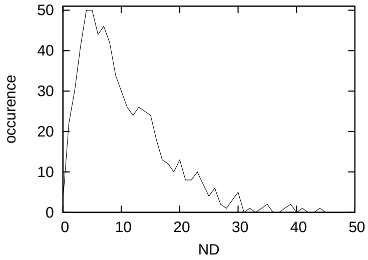
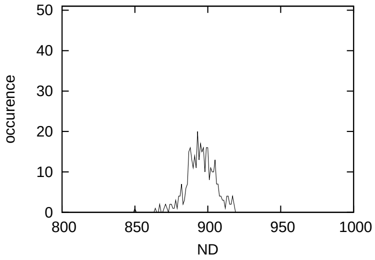
- $N = 1000$
- A simulation was repeated 1000 times for different realizations of J_{ij} distribution.
- time $t = 8$ steps

Case $f \equiv 0$:
Number of defaults (ND) for $J_0 = 0.0001$, $\sigma = 0.001$.

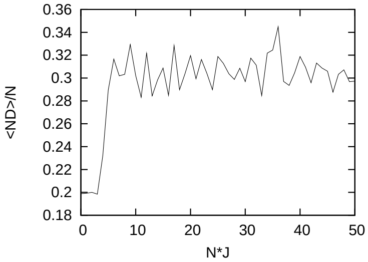


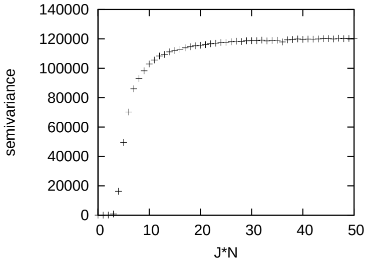
Case $f \equiv 0$:
Number of defaults (ND) for $J_0 = 0.02$, $\sigma = 0.001$.



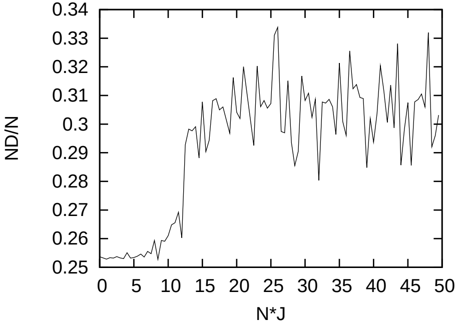


ND as a function of J_0 for $\sigma = 0.001$

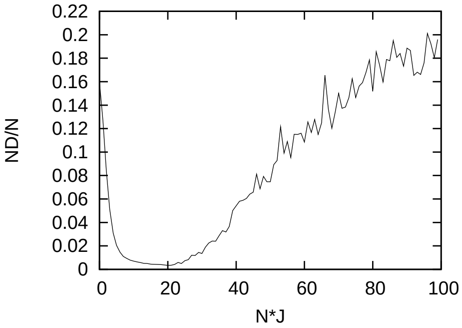


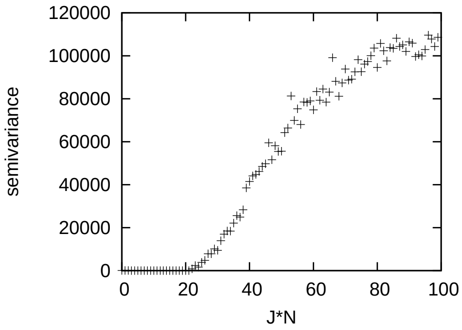


ND as a function of J_0 for $\sigma = 0.2$



$f(R, s)$ depends only on s:
$\exp(f(R, -1)) = 0.15$, $\exp(f(R, 0)) = 0.75$, $\exp(f(R, 1)) = 0.10$.




semivariance
J*N
120000
100000
80000
60000
40000
20000
0
0
20
40
60
80
100

An external field which model panic after a first default:

$$H(R_1, ..., R_N) = H \cdot (1 - \prod_j (1 - \delta(R_j, 0))). \tag{4}$$

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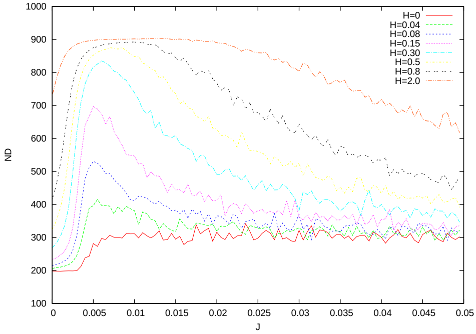
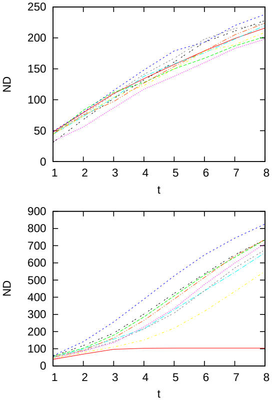
$$H(R_1, ..., R_N) = H \cdot (1 - \prod_j (1 - \delta(R_j, 0))). \tag{4}$$

This field takes a value H if there is at least one bankrupt among all nodes and is equal to 0 in the remaining cases. The modified conditional probability for a variable s_i, $i = 1, 2, ..., N$ is:

$$P(s_i \setminus s_1, ..., s_{i-1}, s_{i+1}, ..., s_N, R_i) = \\ = \frac{1}{Z} \exp(\sum_{j \neq i} J_{ij}\delta(s_i, s_j) + H(R_1, ..., R_N)\delta(s_i, -1)). \tag{5}$$



Number of defaults (ND) as a function of J_0 for different values of the field amplitude H.



Evolution of ND(t) in time for:

- $J_0 N = 0.1,\ H = 0.08$
- $J_0 N = 5,\ H = 0.08.$

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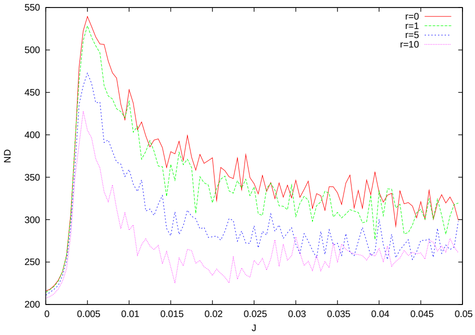

- We used a concept of Potts spin-glass to model collective firm bankruptcies
- There is an optimal value of system coupling for system stability
- The presence of panic field (Lehman Brothers bank effect) is most crucial at the transition point
- Helping the first bankrupts leads to a delay of the global economy collapse

Ref: P. Sieczka, J.A. Holyst, Eur. Phys. Journ. B 71, 461 (2009)
P. Sieczka, D. Sornette, J.A. Holyst, Swiss Finance Institute , Research Paper Series N 10-06, submitted to Eur. Phys. Journ. B (2010)