Cooperation model with costly punishment

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Introduction

Human societies during the process of evolution developed very complex forms of cooperation. From collective hunting in the Stone Age to modern public health care systems - voluntary cooperation is one of the most important indicators of civilization.
In general, cooperation can be described as a process in a society, the members of which are willing to participate, i.e. to pay some cost, in order to maintain some public good. The only purpose of the public good is to bring benefits to its contributors.
However, as soon as any public good forms, the free-riding issue arises. Free-riders are members of the society who are not contributing to the public good but are still drawing benefits from it.
If free-riding becomes the dominant strategy, the public good vanishes. Thus preventing free-riding is a big challenge for societies.
Free-riding problem has been recognized in many areas of our life: avoidance of tax payments or health care insurance premiums, using public transport without paying for tickets, exploiting natural resources, poaching or wireless network leeching.
In order to prevent this issue, various forms of punishing are introduced. Basically, punishing means paying additional cost by contributors to punish the wrong-doers by reducing their payoff. Thus punishment is an altruistic act which helps to keep the public good.

Why yet another model?

In recent years the issue of costly punishment has been a subject of research in the field of game theory. Game-theoretic models assume that strategies spread according to individual rational motivations. Players adopt new strategies in order to maximize their payoff.
However, there is another approach. Opinions, cultures, languages and epidemics can spread in a population as a result of individual personal interactions in the social network. For example, an individual can change their opinion under the influence of social impact or can get infected with a disease from one of his neighbours.

The model

In our model we assume 4 types of transitions of strategies of players according to the interactions with other agents. These transitions are:
α (Freerider -> Contributor) - a freerider changes their strategy to "contribute" after interaction with a punisher with probability α (α is the efficiency parameter).
β (Punisher -> Contributor) - a punisher becomes a contributor with probability β, which is the cost parameter of punishing, after meeting any non-freerider.
γ (Contributor -> Freerider) - a contributor is tempted to freeride, they change their strategy to "freeriding" with probability γ after interactions with other freeriders.
δ (Contributor -> Punisher) - a contributor get irritated by freeriders and change their strategy to "punish" with probability δ after meeting a freerider.

$$\begin{cases} \frac{dC}{dt} = PF\alpha + P(1-F)\beta - CF(\delta+\gamma) \\ \frac{dF}{dt} = CF\gamma - FP\alpha \\ \frac{dP}{dt} = CF\delta - P(1-F)\beta \end{cases} \qquad \begin{aligned} C^* &= \frac{\delta\alpha^2}{\delta\alpha^2+\beta\gamma^2+\alpha\gamma(\beta+\delta)} \\ F^* &= 1 - C^*\left(\frac{\gamma}{\alpha}+1\right) \\ P^* &= \frac{\gamma}{\alpha}C^* \end{aligned}$$

We compare the results from two complementary implementations of our model. The first one is a mean field differential system model. From this model we obtained vector fields using the Euler's method and the value of the stable fixed point. The second approach is a numerical agent-based model of random-walkers on a regular square lattice. From the agent-based model we obtained trajectories for a number of different initial conditions.