Tsallis relative entropy Econometrics to balancing an ecological social accounting matrix of Poland(SAM)2005

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Contents

I.On the generalised pure inverse problems solution

- The question set up in this presentation is the one of generalised pure inverse problem solution.
- The quintessence of the inverse problem
- is conveyed by the expression :
- (1) $y=BX(+e)$.
- Then, given a state X and operator B,what is y.

- Often one must cope with random (gaussian or not) noise or shocks or innovation, denoted above by *e.* In social science analyses, this usually complicates matters. But recent development in classical econometrics and computational capacities have lead to significant progresses.
- Further, much deeper is *the inver*se *question :* given y and a specific B, what is

the true state X?

- If *B* should also be a functional of X, the problem becomes arbitrarily nonlinear.

Physicians are every day coping with such kind of inferring problems. Patients display identical symptoms from different sicknesses. They will need more historical information on the patients to try the solution.

- We will discuss here only the generalized pure linear inverse problem solution.

We then provide the linear integral equation of the first Kind:

- (2) $$y(x) = \int_a^b B(x, y) X(y) dy$$
- Or in familiar discrete form:
- (3) $$y_i = \sum_j B_{ij} X_j$$

- The classical example is the Jaynes dice inverse problem.

II.Maximum entropy principle to solving the inverse problems

- Let us start with a simple case of a dice.
- If a dice is fair and we throw it a very large times n, the expected valued will be 3.5, as from a uniform distribution.

- How to infer about f_i if we have?:
- (4) $4.5 = \sum_{i=1} i f_i$
- Where frequencies *fi* is *ni/n.*

- Which estimate of the set of frequencies would most likely yield this number?

The problem is underdetermined since there is many sets of *fi* that can be found to fit the single datum of equation (4).

Here we have to deal with a multinomial distribution where the multinomial coefficient *w* is given by:

$$W = \frac{N!}{Nf_1!\,Nf_2!..Nf_k!}$$

- In the case of a dice, parameter k equals 6 and w the number yielding a particular

set of frequencies among 6^N possible outcomes.

- We need only find the set of frequencies maximizing w in order to find the set that can be realized in the greatest number of
- ways.
- This is the same logic as maximizing the Shannon Gibbs entropy with output:
- (5) $H(f) = n^{-1} \log w = -\sum_i f_i \log f_i$

- Jaynes(1957, 1984) maximized Shannon function through the restriction of information at hand. This opened entropy theory application to many scientific fields, including social sciences .
- In the case of a dice, the formulation becomes:

  Maximising (5) H(p) = =-F'ln F through the consistency (4), that is the a priori information about the system and the condition fulfillment of f_i additivity to one.

The outputs-using GAMS software, for above presented dice problem is:

(f1,f2,f3,f4,f5,f6)=

(0.054, 0.079, 0.114, 0.166, 0.240, 0.348).

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The next chapters generalize Shannon-Gibbs-Jaynes maximum entropy principle to Kullback-Leibler relative entropy. Next, the last targeted presentation will deal with Tsallis power law distribution to generalize Kullback-Leibler cross entropy.

III.Cross entropy principle

- Kulback (1957) et Good (1963) extended the Janes Shannon Gibbs model by formulating the principle of minimum cross(relative) entropy. Using an a priori information Q about unknown parameter P, the resulting formulation is as follows:
- (6) $\text{MinI}(p,q) = \sum pk\ln(pk/qk) = P'\ln P - P'\ln Q$
- (7) $Y = XP$
- (8) $P'1 = 1$
-

- One should note that when Q is fully consistent with moments, then P=Q and the distribution becomes uniform with qk = 1/K.This leads to the solution of maximum entropy principle.
- Thus, cross entropy principle stands for a generalization of the one of maximum entropy.

IV.Tsallis relative entropy extensions

- Like in statistical physics, socio-economical random events display two types of behavior: ergodic and non ergodic systems.
- Whenever isolated in a closed space, ergodic systems dynamically visit with equal probability all the allowed micro states(Constantino Tsallis,2004) .

- Following Tsallis(2004), it seems logic to imagine systems visiting the allowed micro states in much more complex way than defined by ergodicity.

- We generalize the equation (6) by the Tsallis relative entropy formulation.

- If we use in the next equations usual symbols of social accounting matrix, a general definition of Tsallis maximum

- entropy is presented as follows:

$$S_q(A_{ij}) = -\sum_{k=1}^{n} A_{ij}{}^{q} \ln_q A_{ij}$$

- And the Tsallis relative entropy as:
- (9)

$$I_q(A_{ij} // \vec{A}) = -\sum_{k=1}^{n} A_{ij} \ln_q \frac{\bar{A}}{A_{ij}}$$

- The latter formula measures the discrepancies between the a priori $\bar{A}$

(i.e. the not balanced SAMEA) and the posteriori A_{ij} (i.e. the balanced SAMEA) .

V.A SAM as a Walrasian equilibrium framework

- A social accounting matrix(SAM) is a square table which presents a summary of territorial(national) economic activities usually for a one year period.

- The underlying principle of SAM accounting is double entry bookkeeping by which revenues of any account are depicted along its row whereas expenditures of the same account are shown along its column.

Balancing a SAM

- Under neoclassical warlasian hypotheses, the above accounting must balance. Warlasian equilibrium is based upon **the law of product and of value conservation.**
- Conservation law of product: while *ensuring that the flows of goods and factors must be absorbed by the production and consumption activities in the economy, is an expression* ***of the principle of no free disposability*** .

Product and value laws of conservation = Cornerstones of warlasian equilibrium

- Conservation of product, while *ensuring that the flows of goods and factors must be absorbed by the production and consumption activities in the economy, is an expression* ***of the principle of no free disposability*** or ***market clearance***.

Conservation law of value

- Conservation of value implies that the sum total of revenue from the production of goods must be allocated to rented primary factors, to other industries for intermediate inputs, or to the government as taxes.
- This industries make ***zero profit*** .This law simultaneously suggests *constancy of returns to scale* in production, *perfectly competitive markets* for produced commodities and balanced budget.

Outputs of SAMEA balancing

- We have used the software GAMS to balance the Polish social accounting matrix (2005) with ecological aspects.
- Annex 1b presents the no balanced SAMEA, and annex 2 presents the post Tsallis entropy balanced matrix.
- Data source: http://www.stat.gov.pl/gus/

Concluding remarks

- According to us, interesting and promising multidisciplinary approach.
- Better to generalise the Shanon entropy by the non extensive entropy approach which better tackles the problem of heavy queue when normal distribution is put into question.Data recovered from surveys desserve such attention.

Annex 1.a. Macroeconomic structure of a SAMEA

Activities	Commodities	Capital	Enterprises	Households	Public institutions	Rest of the world	Saving netto	Natural resources	Environment	Total(income)
	C			Self consumptions				Environment Protection	Pollutants Emission	Total production(net o
c				c	c	export				Total sale
W+Profits										W+Profits
axes				C	G_F	$(X-M)_K$	I			$C+G_F+(X-M)_K+I$
		W+Profits			G_H	$(X-M)_C$		Environment Protection	Pollutants Emission	$W+G_H+(X-M)_C$
	consumption taxes		T_F	T_H				Environment Protection	Pollutants Emission	T_F+T_H
	imports		$(X-M)_K$	$(X-M)_C$						$(X-M)_K+(X-M)_C$
				S_H	S_G					S_H+S_G
ction process										Natural resource use
ance absorption						Cross border pollutants				Environment de
costs	Total Absorption	W+Profits	$T_F+(X-M)_K$	$C+T_H+(X-M)_C+S_H$	$G_F+G_H+S_G$	$(X-M)_C+(X-M)_K$	I	Environment Protection	Pollutants Emission	

Annex 1b. The SAMEA 2005 of Poland

	Activity	Commodities	Labor	Capital	Households	Enterprises	GVMNT	Nonprofit Inst	CapAccount	NatRes	Envirnmt	RoW
Activity		1906696033			47186003						6197,5	
Commodities	1055366158											
Labor	352183426		0	0	0	0		0		0	0	
Capital	505346700		0	0	0	0	0	0	0	0	0	0
Households			3,52E+08	0	0	14927790	153946000	0	0	0	17361,2	0
Enterprises			0	505346700	2828976	6658576	20901000	3862000	0	0	0	27013000
GVMNT	40985752	84786414	0	0	187938000	24492000	13000	0	0	0	7811,4	0
NonprofitInst*			0	0	0	0	1022	0	0	0	0	0
CapAccount			0	0	500453	603456	400878	2023	0	0	0	45000
NatRes			0	0	2809911	0	8113	10952	872267	7606	0	5455412
Envirnmt			0	0	1410045	0	0	0	0	0	0	0
RoW		358825380	0	0	0	1460000	11607000	0	0	0	0	0
Total	1953882036	2350307827	3,52E+08	505346700	242673388	48141822	186877013	3874975	872267	7606	154100	32513412

- Macsam1 Assigned new balanced SAM flows from TCE X10 000 000

	Act	Com	Labor	Capital	Hou	GRE	NonprInst	Cap	NatRes	Envirnmt	RoW	Total
Act		191.841			3.487				0.224	0.237		195.789
Com	98.548				59.047	26.478		6.587			44.357	235.018
Labor	31.640											31.640
Capital	49.164											49.164
Hou			31.640	49.164					0.144	0.123		81.072
GRE	16.436	1.524			5.690		0.559				2.849	27.058
NonprInst						0.565						0.565
Cap					10.179		0.005				0.395	10.579
NatRes					0.368		0.001					0.369
Envirnmt					0.360							0.360
RoW		41.654			1.942	0.014		3.992				47.601
Total	195.789	235.018	31.640	49.164	81.072	27.058	0.565	10.579	0.369	0.360		

Annex 3. Ecological aspects with impact on sustainable development

Resource	**Water**	Freshwater occupancy per capita (m^3)
		Usage efficiency of freshwater (Urban vs. Rural)
		Ratio of underground water exploitation
	Land	**Arable land per capita** (hectare)
		Multicropping index
	Forest	Forestry coverage rate
		Wood occupancy per capita (m^3)
	Ocean	Ratio of breed aquatics
		Fishing aquatics rate
	Pasture	Pasture per capita (hectare)
	Ore	Mining efficiency of ores
		Ratio of mined ores
	Energy	**Energy intensity index**
		Ratio of clean energy
	Waste	Ratio of waste recycling
Environment	**Water**	**Water pollution index**
	Land	Ratio of deserting land
		Ratio of water and soil erosion
	Air	**Air pollution index**
	Waste	Industrial waste treatment rate
		Urban garbage treatment rate
	Noise	Urban noise index
	Biodiversity	Ratio of threatened species
	Protection	Urban greenbelt per capita (m^2)
		Ratio of “green” products

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- Dziekuje za uwage