Microeconomic evolution model with technology diffusion

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Presentation outline

1. Introduction

2. Model setup

3. Model properties

4. Conclusions

Introduction

- The model is a modification of:

  M. Ausloos, P. Clippe, A. Pękalski, *Model of macroeconomic evolution in stable regionally dependent economic fields*, Physica **A 337**, 269-287; arXiv:cond-mat/0401144.

- From the abstract of this paper:
  We develop a model for the evolution of economic entities within a geographical type of framework. On a square symmetry lattice made of three (economic) regions, firms, described by a scalar fitness, are allowed to move, adapt, merge or create spin-offs under predetermined rules, in a space and time dependent economic environment. We only consider here one timely variation of the "external economic field condition". For the firm fitness evolution we take into account a constraint such that the disappearance of a firm modifies the fitness of nearest neighboring ones.

Introduction

Key modifications:

- different context – **fitness of firms** interpreted as their **level of technology**
- one economic region instead of three
- **external economic field** interpreted as **frontier technology** and changes continuously, instead of one change of value ($\Rightarrow$ notion of **technological backwardness**)
- under certain circumstances the level of technology can increase via interaction with technological frontier
- modified probability of firm's survival
- additional variable that describes firms – **firm size**

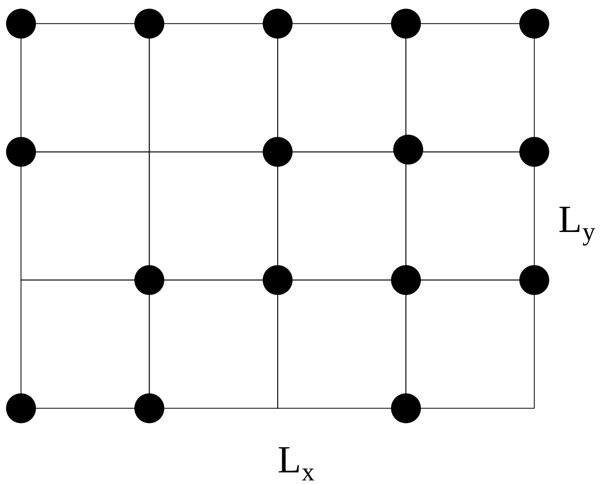
Simulational models

- The idea of simulational models is to introduce heterogenous agents and rules of their interaction.
- These rules can be described as probabilities of some behaviour in a given situation.
- probabilities $\longrightarrow$ random numbers $\longrightarrow$ Monte Carlo simulation.

Model setup

- We consider a square lattice of $L_x \times L_y$ sites.
- Each site can be occupied by one or zero firms.
- The initial concentration of firms is c_0 and the number of firms at a given time is denoted by $N(t)$.

$$c_0 = \frac{N(0)}{L_x L_y} \tag{1}$$



Model setup

- Each firm is characterized by 2 variables:
  - $A_i(t)$ – technological advancement,
  - $w_i(t)$ – firm's size (weight), i.e. their relative market share ($\forall t \ \sum_i w_i(t) = 1$).

- At $t = 0$, A_i are uniformly distributed in the interval $(0, A_{max})$, hence:

$$\langle A(0) \rangle \approx 0.5 A_{max}. \tag{2}$$

- The initial weights are:

$$w_i = \frac{1}{c_0 L_x L_y} \quad \forall i. \tag{3}$$

Model setup

- Firms can move on the lattice, disappear, merge and create spin-offs.

- The probability of survival of a firm depends on their distance from technological frontier.

- The dynamics of technological frontier is assumed to be:

$$F(t) = e^{\sigma t}, \tag{4}$$

  where σ is an exogenous parameter ("world" technological progress).

- The parameter s measures the sensitivity to technological backwardness.

- The minimal number of firms is min (the number of firms can not go below min).

Simulation algorithm – firm's survival

The system evolves according to the following rules:

1. Randomly choose a firm i from among the total number of $N(t)$ firms.

2. Calculate the probability of this firm's survival:

$$p_i = \begin{cases} e^{-s(\langle A(t)\rangle F(t) - A_i(t))} & \text{gdy } \langle A(t)\rangle F(t) > A_i(t),\ \langle A(t)\rangle < 1 \\ e^{-s(F(t) - A_i(t))} & \text{gdy } F(t) > A_i(t),\ \langle A(t)\rangle \geq 1 \\ 1 & \text{gdy } A_i(t) \geq \langle A(t)\rangle F(t),\ \langle A(t)\rangle < 1 \\ 1 & \text{gdy } A_i(t) \geq F(t),\ \langle A(t)\rangle \geq 1 \end{cases} \quad (5)$$

Simulation algorithm – outer technology diffusion

3. Draw a random number r from the uniform distribution on the interval $[0, 1]$.

   If $r > p_i$, the firm disappears and its lattice site becomes empty. The weights of remaining firms are adjusted accordingly (such that the normalization condition $\sum_i w_i(t) = 1$ holds. The algorithm returns to point 1.

   If $r \leq p_i$, the firm tries to move one lattice spacing away. Draw a random number r_1 . If $r_1 < 0.25$, we check whether the "site to the North" is empty. If $0.25 \leq r_1 < 0.5$ the "site to the West" is checked etc.

4. If the neighbouring site is empty, the firm moves to this site and checks whether there is some other firm in the nearest neighbourhood of the new site.

   If such firm is absent, the firm profits from **outer technology diffusion**, according to the formula:

$$A_i(t) \to A_i(t) + r_2(F(t) - A_i(t)), \tag{6}$$

   where r_2 is a number drawn from the uniform distribution. The algorithm returns to point 1.

Simulation algorithm – inner technology diffusion (firms' merger)

5. If there is some firm j in the neighbourhood of the firm i, then:

   - with probability b (which is a parameter of the model), the firms merge. The technology of the new firm is:

$$\begin{aligned} &\text{VAR. 1:} \quad A_i(t) \to 0.5(A_i(t) + A_j(t) + 0.5 r_3 |A_i(t) - A_j(t)|), \\ &\text{VAR. 2:} \quad A_i(t) \to \max\{A_i(t), A_j(t)\}, \end{aligned} \tag{7}$$

   where r_3 is a number drawn from the uniform distribution. The firm j disappears from the system and the weight of the new firm is equal to the sum of weights of the merging firms. In the first variant, the technology of the new firm is the arithmetic mean of the technology levels of the merging firms plus some synergy effect, which is larger if the difference in technologies is larger. In the second variant, the new firm's level of technology is equal to the technology level of the technologically more advanced firm The synergy effects will be called the **inner technology diffusion**.

Simulation algorithm – inner technology diffusion (spin-off)

5. If there is some firm j in the neighbourhood of the firm i, then:

   - With probability $1-b$, the firms i and j create a spin-off. A firm k emerges and it is located in the 8-site neighbourhood of the firm i (north, north-west, west, south-west etc.). The positioning procedure is analogous to the one in point 3 (a number r_4 is drawn and depending on the outcome a suitable site is chosen). If the appropriate site is not empty, the spin-off does not emerge. The technology of the spin-off equals:

$$\begin{aligned} &\text{VAR. 1:} \quad A_i(t) \to 0.5(A_i(t) + A_j(t) + 0.5r_5|A_i(t) - A_j(t)|), \\ &\text{VAR. 2:} \quad A_i(t) \to \max\{A_i(t), A_j(t)\}, \end{aligned} \tag{8}$$

   where r_5 is a number drawn from the uniform distribution. Hence, the economy profits from **inner technology diffusion** also if a spin-off is created. The weight of the new firm is equal to the sum of weights of firms i and j, multiplied by a parameter $w_s \in [0,1]$. The weights of firms i and j decrease accordingly, by $w_i w_s$ and $w_j w_s$.

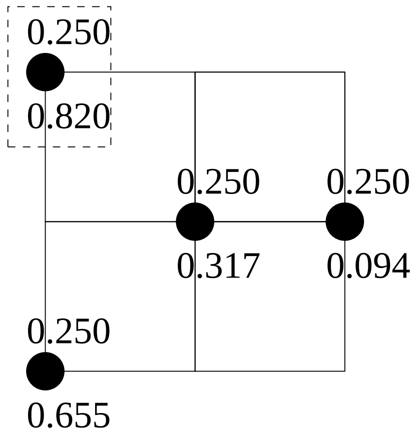
Simulation algorithm

6. We return to point 1 of the algorithm until $N(t)$ firms have been chosen. Then, a Monte Carlo step is finished, i.e. we set $t \longrightarrow t+1$. The random choice of a firm in point 1 of the algorithm implies that a given firm can be chosen more than once at time t and hence firms which are not chosen at this Monte Carlo step exist.

Example of a simple simulation

- $L_x = L_y = 3$ – 3×3 lattice,
- $c_0 = 4/9 \Rightarrow N(0) = 4$ firms,
- $\sigma = 0.01$ – technological progress rate (growth rate of technological frontier),
- $s = 1.0$ – sensitivity to technological backwardness,
- $b = 0.1$ – probability of firms' merger,
- $min = 2$ – minimal number of firms,
- $A_{max} = 1$ – the maximal level of technology at $t = 0$,
- $w_{spin-off} = 0.1$ – the ratio of spin-off's size to parent firms' size.

Initial configuration



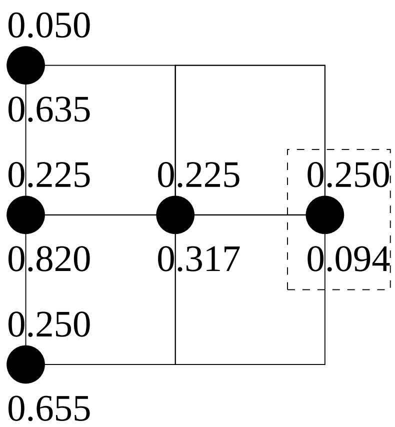
$$\langle A \rangle = 0.4715$$

$$F = 1$$

$$\langle \tilde{A} \rangle = 0.4715$$

Firm (1,1) — $p = 1$, $r_1 = 0.226 \Rightarrow$ north.
$r_b = 0.902 \Rightarrow$ Firms (2,1) and (2,2) create a spin-off.
Technology of the spin-off: $0.820 + 0.317 \longrightarrow 0.635$.

Second firm



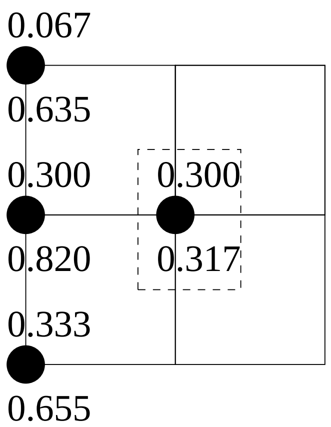
$$\langle A \rangle = 0.4748$$

$$F = 1$$

$$\langle \tilde{A} \rangle = 0.4748$$

Firm (2,3) — $p = 0.683$, $r = 0.701$ $\Rightarrow$ firm disappears.

Third firm



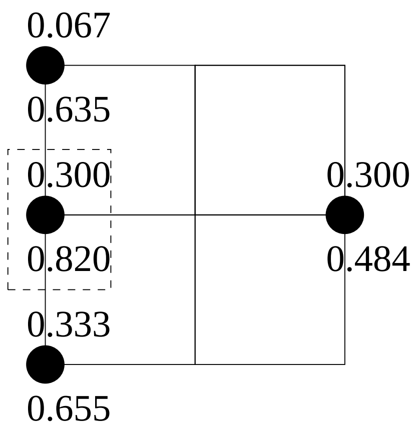
$\langle A \rangle = 0.6018$

$F = 1$

$\langle \tilde{A} \rangle = 0.6018$

Firm (2,2) — $p = 0.752$, $r = 0.123$, $r_1 = 0.377 \Rightarrow$ west.
Technology diffusion to firm (2,3).
$A_{(2,3)} = 0.317 \rightarrow 0.484$.

Fourth firm



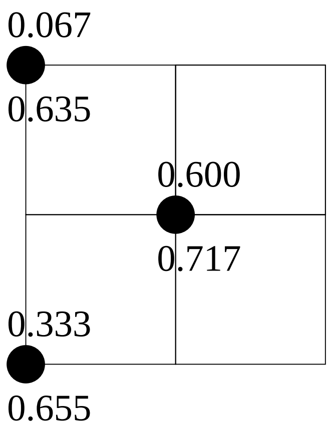
$$\langle A \rangle = 0.6519$$

$$F = 1$$

$$\langle \tilde{A} \rangle = 0.6519$$

Firm (2,1) — $p = 1$, $r_1 = 0.368 \Rightarrow$ west.
$r_b = 0.003 \Rightarrow$ Merger of firms (2,2) and (2,3).
Technology of the new firms: $0.820 + 0.484 \longrightarrow 0.717$.

The end of 0. Monte Carlo step

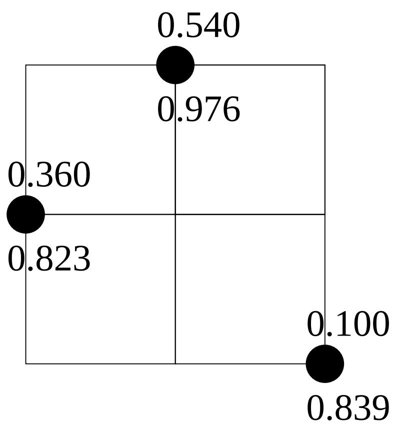


$$\langle A \rangle = 0.6909$$

$$F = 1$$

$$\langle \tilde{A} \rangle = 0.6909$$

After 10 Monte Carlo steps



$$\langle A \rangle = 0.9072$$

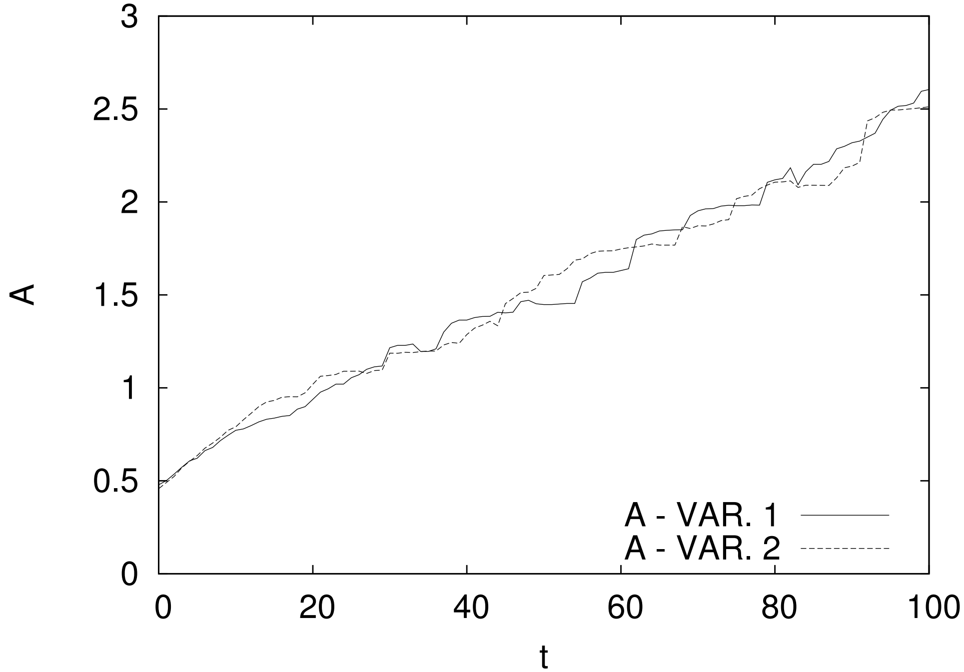
$$F = e^{0,01 \cdot 10} \approx 1.1052$$

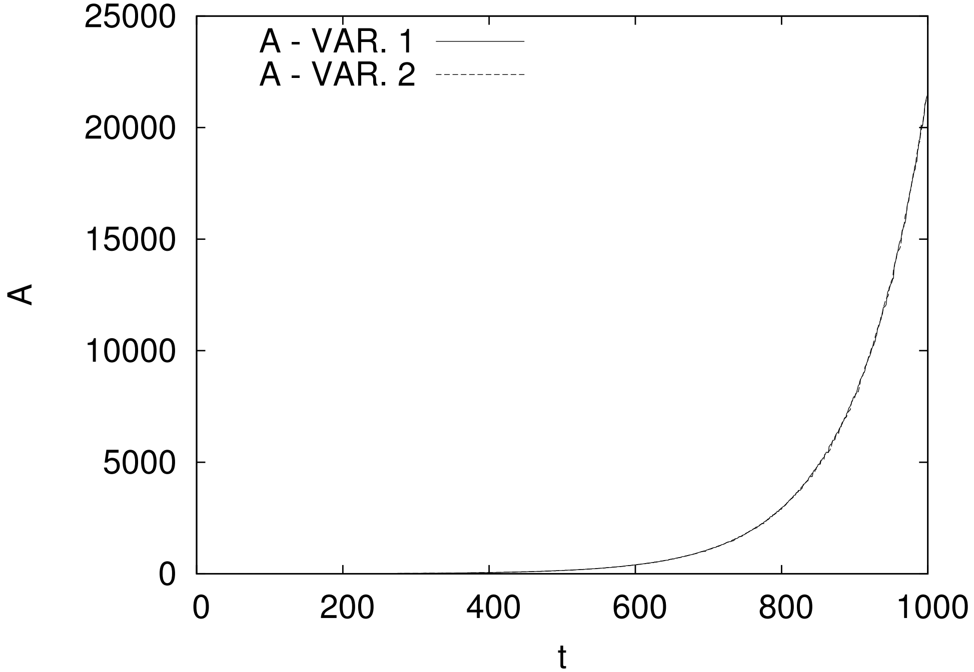
$$\langle \tilde{A} \rangle = 0.8209$$

Parameters for simulations

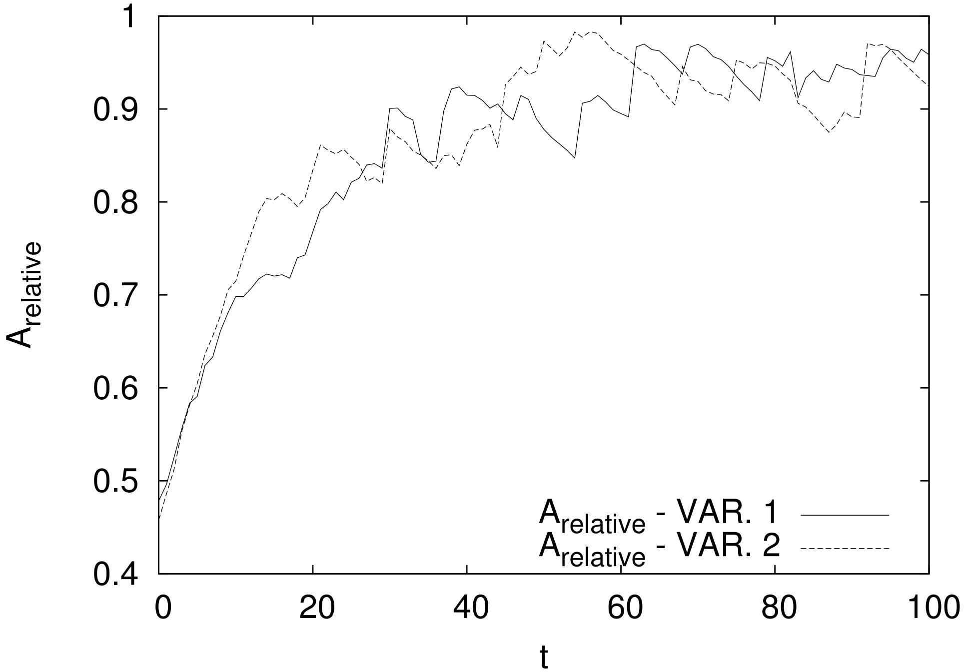
- $L_x = L_y = 10$ – 10×10 lattice,
- $c_0 = 0.8 \Rightarrow N(0) = 80$ firms,
- $\sigma = 0.01$ – technological progress rate (growth rate of technological frontier),
- $s = 1.0$ – sensitivity to technological backwardness,
- $b = 0.01$ – probability of firms' merger,
- $min = 10$ – minimal number of firms,
- $A_{max} = 1$ – the maximal level of technology at $t = 0$,
- $w_{spin-off} = 0.1$ – the ratio of spin-off's size to parent firms' size.

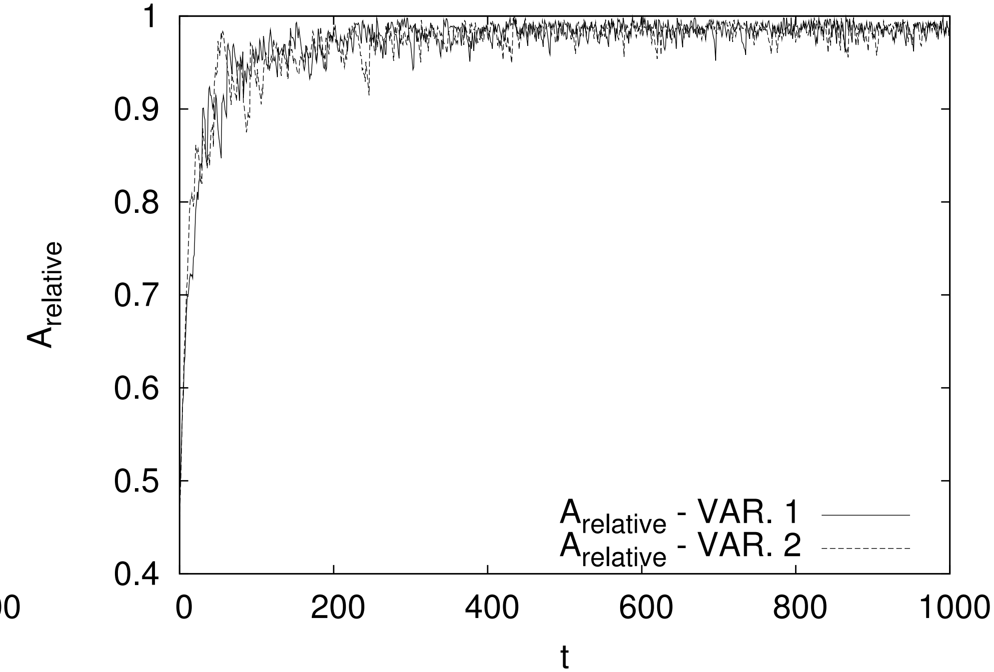
Dynamics of technology in mid- and long-term



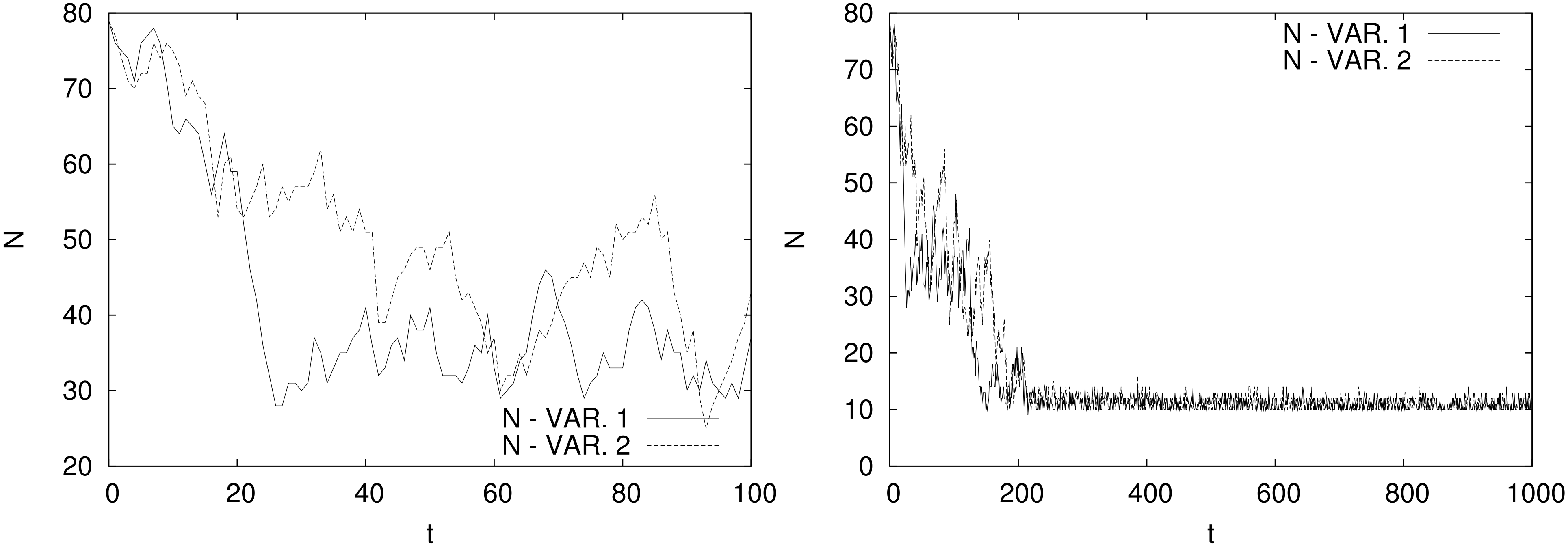


Dynamics of relative technology in mid- and long-term

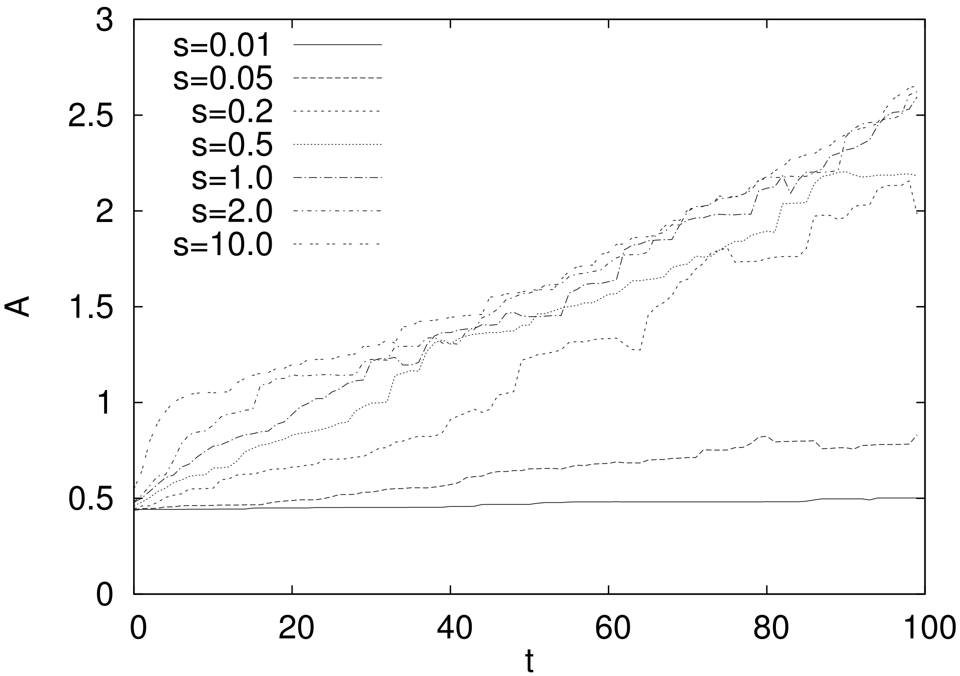


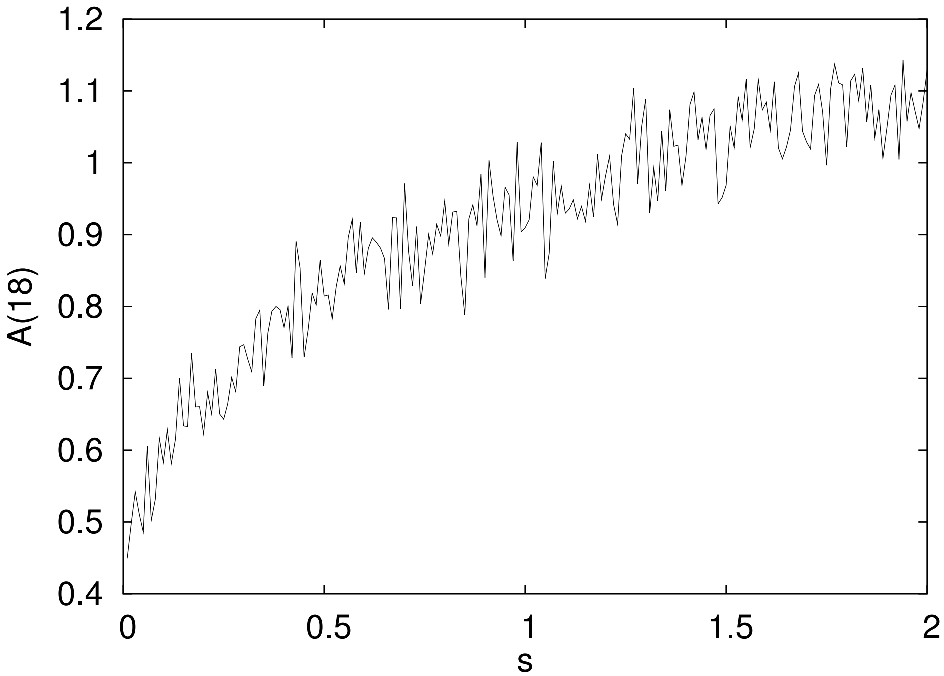


Dynamics of the number of firms in mid- and long-term

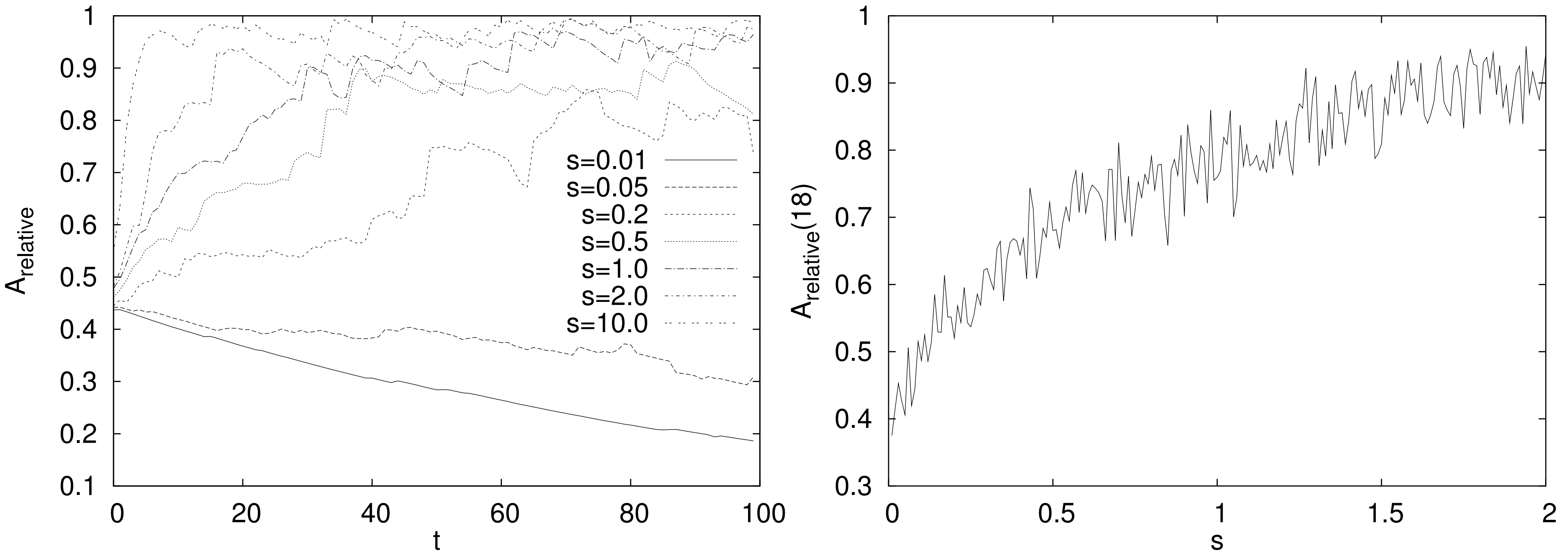


Dynamics of technology – dependence on sensitivity to technological backwardness (the parameter s)





Dynamics of relative technology – dependence on sensitivity to technological backwardness (the parameter s)



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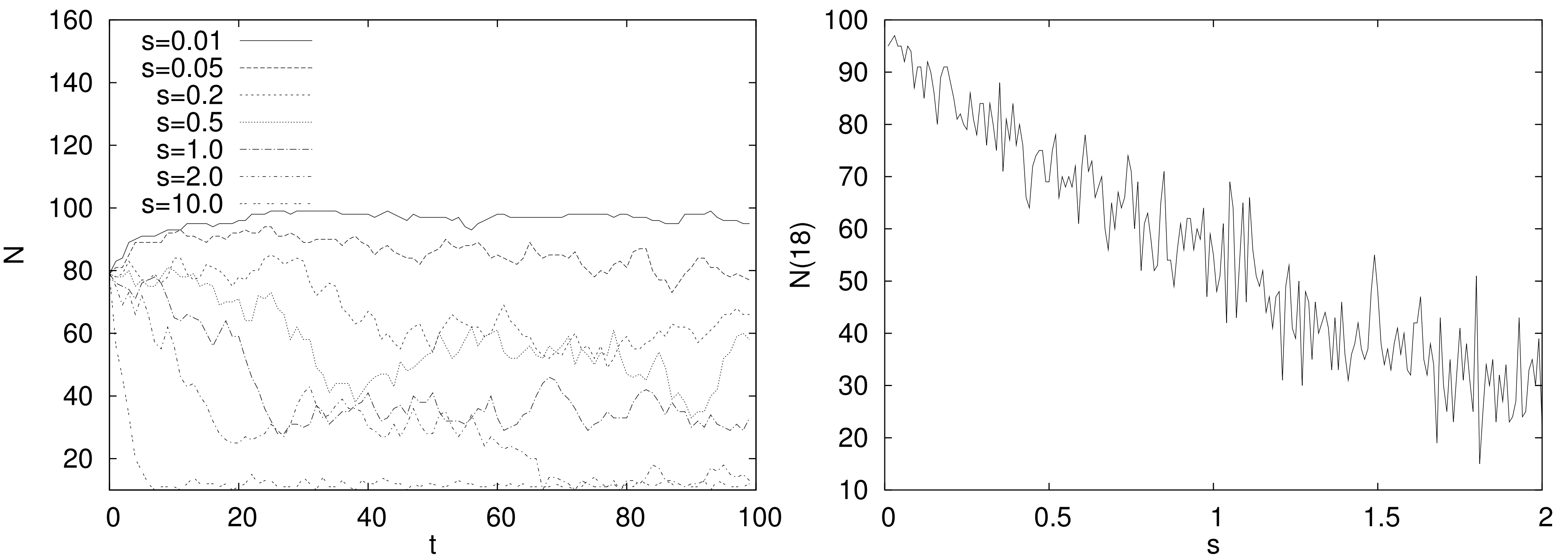


Empirical analysis

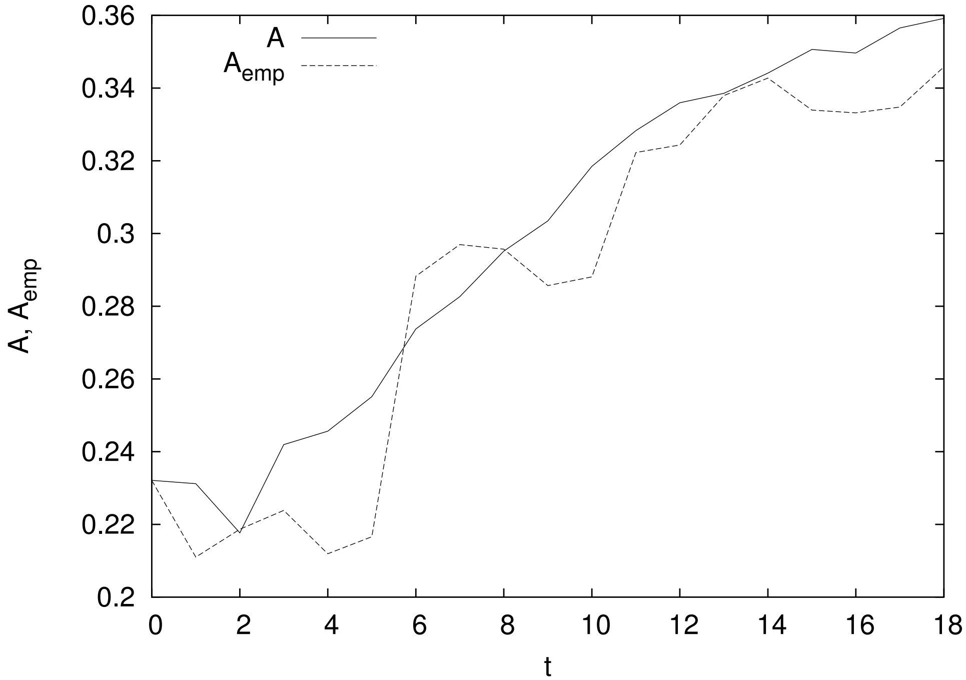
- group of 29 OECD countries (Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Japan, Korea, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Spain, Sweden, Switzerland, Turkey, United Kingdom, United States)

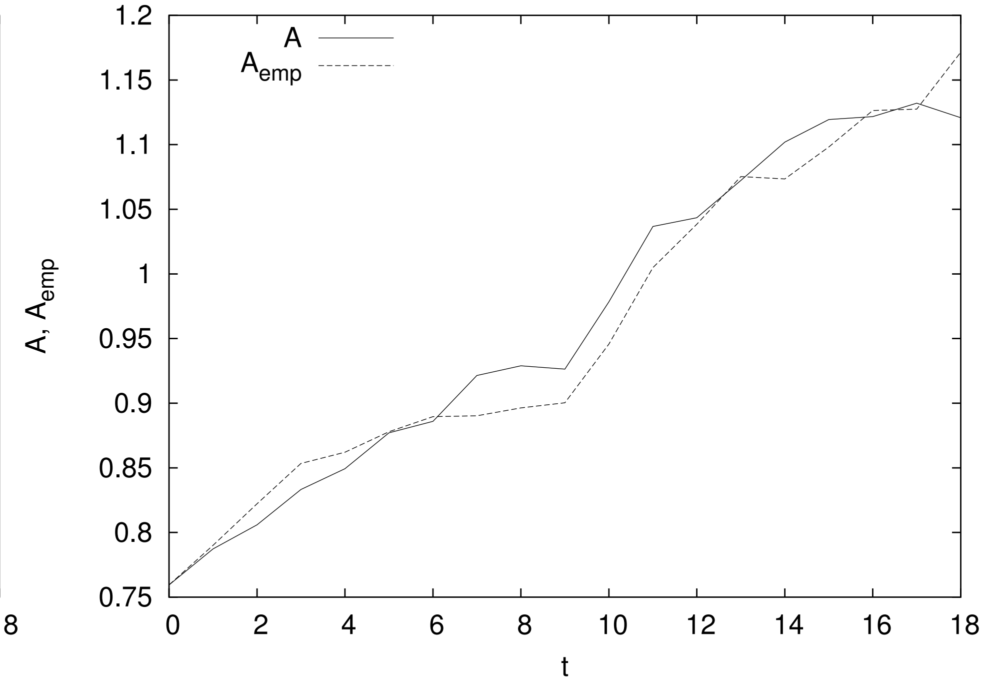
- period of analysis 1981-1999

- technological frontier growth rate (USA technology growth rate) – $\sigma = 0.0208$.

- for each country we calibrate the value of the parameter s.

Example – Poland and Belgium)





Calibrated values of s for OECD countries

C'try	s	MRE	C'try	s	MRE	C'try	s	MRE
AUS	0.80	2.17	GRE	0.58	2.77	NZL	0.46	2.63
AUT	0.70	3.39	HUN	0.02	8.21	NOR	1.73	4.05
BEL	1.46	1.86	IRL	1.38	4.22	POL	0.73	5.37
CAN	1.87	6.86	ISL	1.29	3.29	POR	0.74	5.61
CZE	0.55	5.77	ITA	1.66	2.53	SPA	0.32	3.69
DEN	1.34	3.46	JPN	0.48	2.25	SWE	1.15	2.73
FIN	0.97	2.81	KOR	1.38	9.95	SWI	1.98	2.65
FRA	1.69	3.04	MEX	0.39	6.32	TUR	0.36	2.82
GER	0.94	2.84	NED	0.83	2.04	UK	1.36	3.28

We can distinguish three groups of countries:

- countries in which the inner technology diffusion dominates (low values of s),
- countries in which the inner and outer technology diffusion play a similar role ($s \approx 1$),
- countries in which the outer technology diffusion or autonomous innovations dominate (high values of s).

Three groups of countries

- 1st group – Austria, Czech Republic, Greece, Spain, Japan, Mexico, New Zealand, Poland, Portugal, Hungary and Turkey.⇒ mainly developing countries, in which the technological advancement is **not** the most important factor that determines the probability of survival of firms (the sensitivity to technological backwardness is small).

- 3rd group – Belgium, Canada, Denmark, France, Iceland, Ireland, Italy, S. Korea, Norway, Switzerland, United Kingdom.⇒ mainly highly-developed countries. In such countries, with long traditions of free market economy, the level of technology is one of the most important factors of competition between companies – thus the underdeveloped firms do not survive for long.

- 2nd group – Austria, Finland, Germany, The Netherlands, Sweden.⇒ interpolates between the two other groups. The case of Germany, a country which was divided into two independent states for half of the analyzed period, seems to correspond well with this interpretation.

Conclusions

- The general conclusion that can be drawn from this model is that the sensitivity to technological backwardness is rather small in the developing countries and rather high in the highly-developed states.

- Thus, it is probable that the mechanisms of technological progress are quite different in these groups of countries.

- In the developing countries, technological progress consists mainly in technology transfers from highly-developed companies (which can be interpreted as e.g. firms with international capital) to the underdeveloped ones.

- In the highly-developed countries, in turn, companies' technology levels are more close to one another and the dominating mechanism of progress is the development of autonomous innovations or the use of the most developed technologies available in the world.

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Thank you for your attention!