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On the Zipf strategy for short-term investments in WIG 20 futures

Bartosz Bieda[(a)], Paweł Chodorowski[(b)(c)], Dariusz Grech[(c)]

(a) Wrocław University of Technology, Institute of Telecommunication and Acoustic, Wybrzeże Wyspiańskiego 27, 50-370 Wrocław

(b) Bank Zachodni WBK SA, Finance Division, Rynek 9/11, 50-950 Wrocław

(c) University of Wrocław, Institute of Theoretical Physics, M.Born Sq. 9, 50-204 Wrocław

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Plan of talk

1. Zipf law and its origin, connection with complexity
2. Projection of time series data into sequence of states (spin-like 1dim chain)
3. Zipf exponents for real WIG20 data and for shuffled signals
4. What is WIG20 futures?
5. Details of Zipf strategy applied to WIG futures
6. Conclusions

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Zipf law and its origin

1916 – observation: word frequences in French text are inversely proportional to its index (rank)

Jean-Baptiste Estoup, Gamme Stenographique

$W_K = (k = 1, \ldots, N)$ - different words in given text

$f_K = f(W_K)$ - frequency of the W_K apperance in given text

$\{W_1, W_2, \ldots, W_K, \ldots, W_N\}$ - sequence ordering words

$$i < j \iff f_i > f_j \quad (i, j = 1, \ldots, N - rank)$$

$$f_k \sim k^{-1}$$

1935 – analogous observation in English text,

Georg K. Zipf, The Phsychology of Language

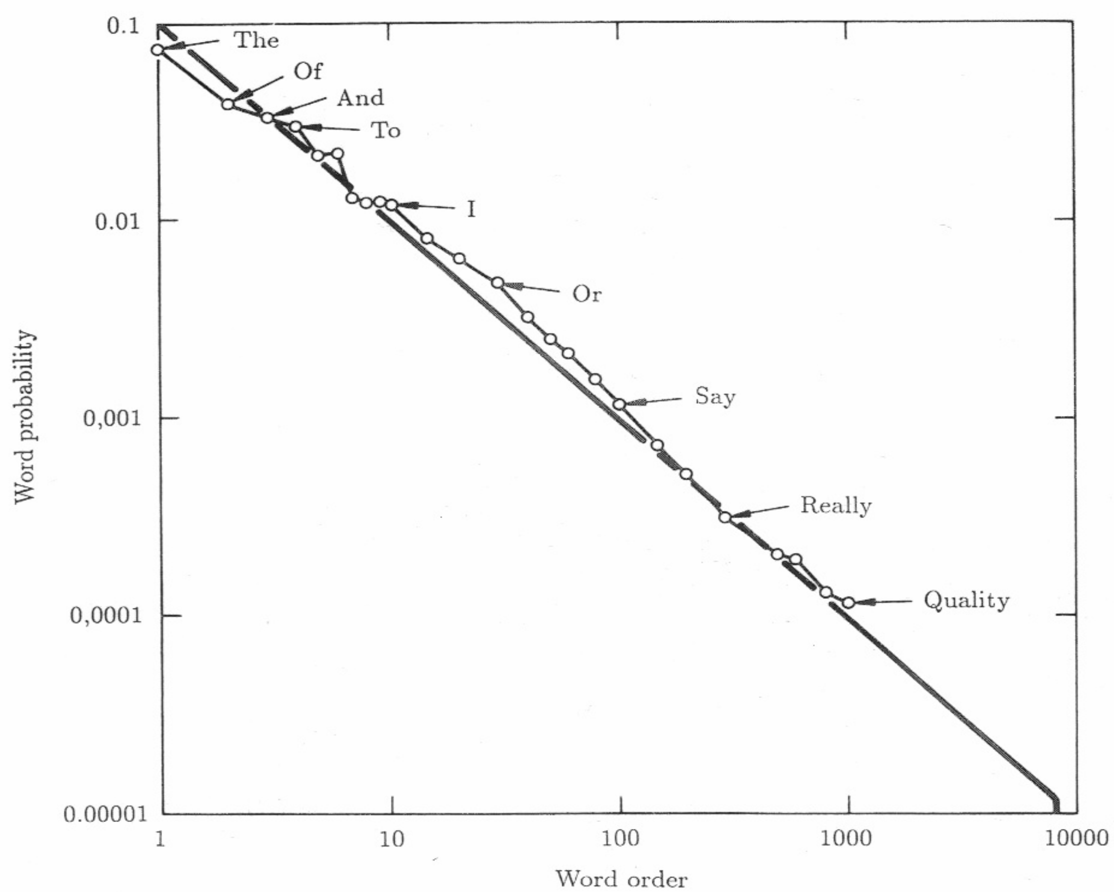
1949 – Zipf power law,

Georg Zipf, Human Behavior and the Principle of Last Effort

$$f_k \sim k^{-\xi}, \ \xi \text{ - Zipf exponent } (\xi > 0)$$

Zipf law and its origin

- Zipf plot for the book „The War of the Worlds” of H. G. Wells
- Some „typical” words are marked



$$f(r) \sim r^{-\alpha}$$

Fig. 1. Zipf plot for the book „The War of the Worlds” of H. G. Wells

Zipf law and its origin

- Zipf plot for the book „Lalka” (The Doll) of B. Prus (Polish)

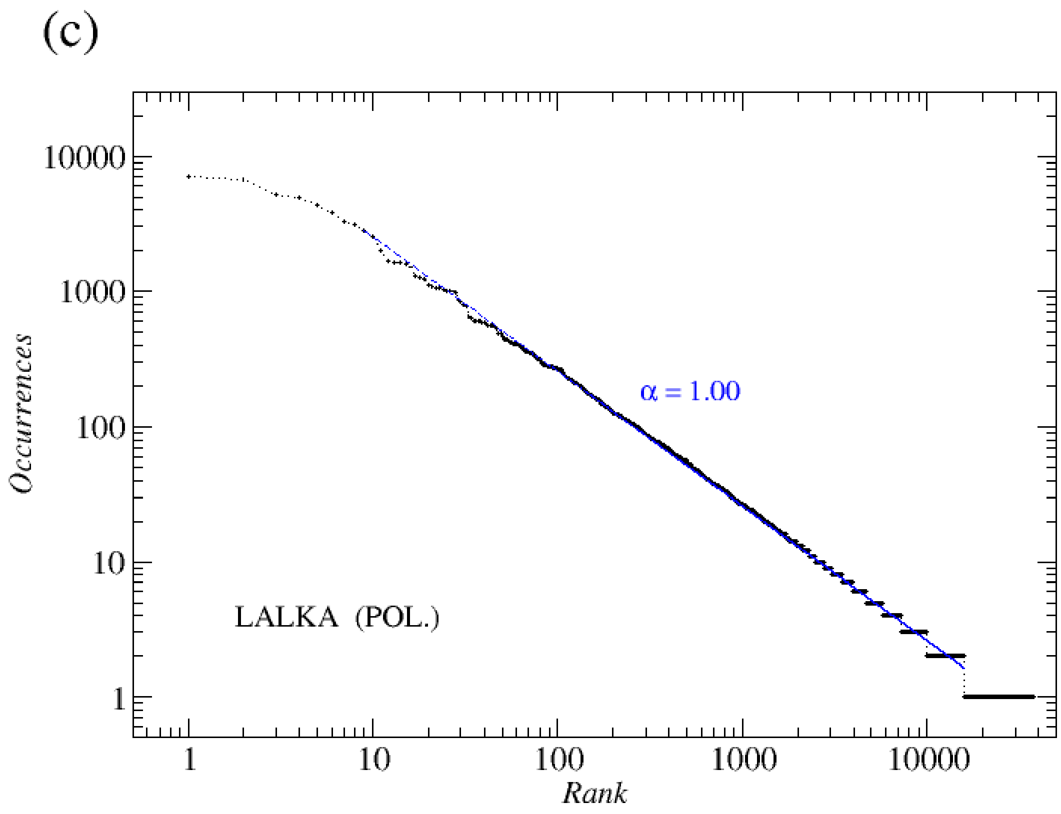


Fig. 2. Taken from S. Drożdż, et. al., Approaching the linguistic complex. ar Xiv: 0901.3291 (2009)

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Zipf law and its origin

- Zipf power-law discovered in many areas of science and arts
  - Literature
  - Biology
  - Economy
  - Finances
  - Geology
  - Genetics
  - Physics

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General formulation of Zipf law

- Ω – set of events $\{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_N\}$ in given system ordered in such way that

$$i < j \iff p(\varepsilon_i) > p(\varepsilon_j); \quad p - probability, \quad i, j = 1, \ldots, N$$

- Zipf power law takes place if $\exists \xi > 0$

$$p(\varepsilon_k) \sim k^{-\xi}$$

Source of Zipf law

• Correlations in complex system (long memory in time series)			Zipf power law
$H > ½$	→		$\xi > 0$
• Criticality (SOC)			
$H \to 1$	→		$\xi \to 1$

$$\boxed{\xi = 2H - 1}$$

A. Czirok, R.N. Mantegna, S. Havlin, H.E. Stanley, Phys. Rev. E52 (1995), 446.

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COLLECTION OF TEXTS - CORPUS

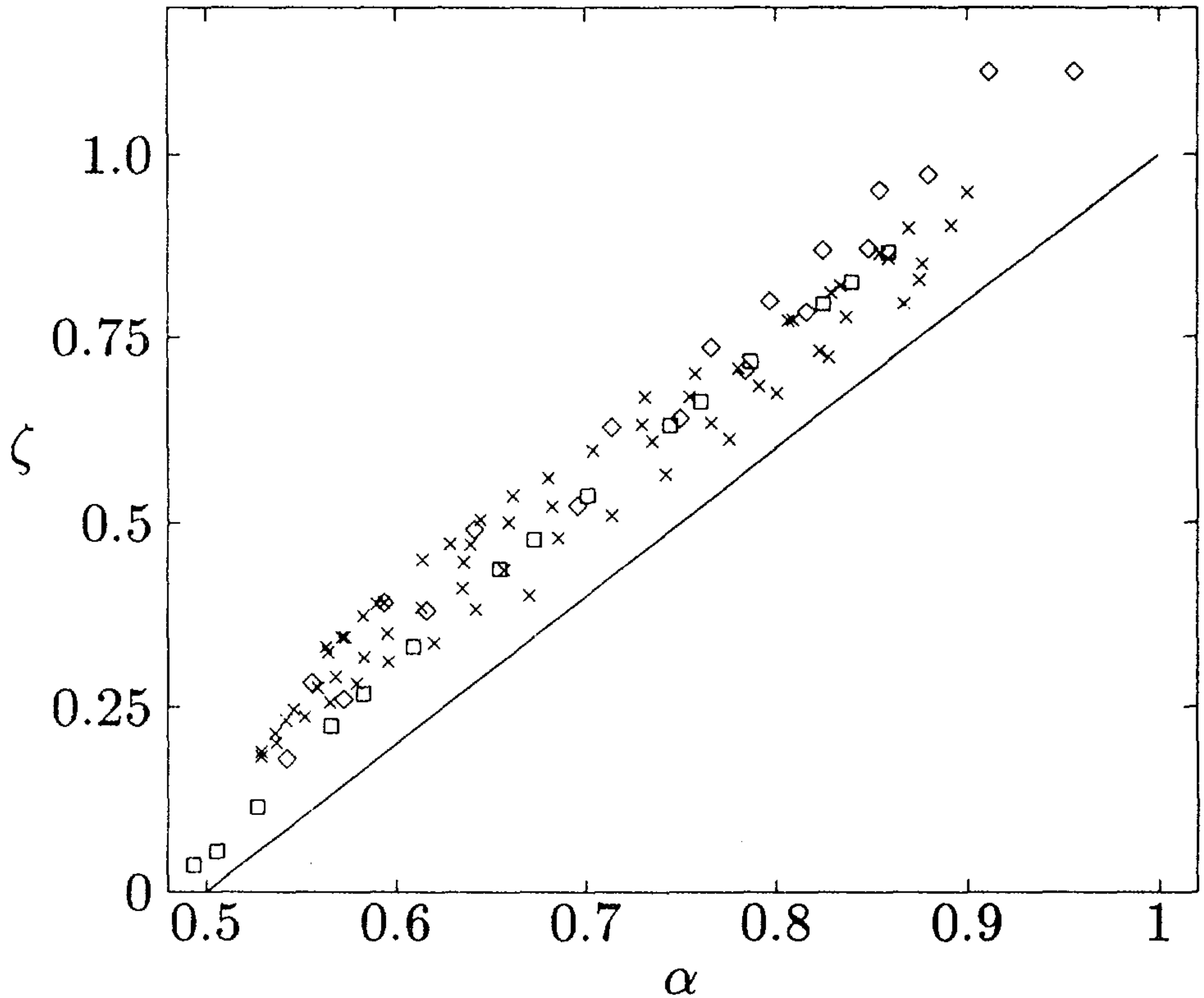

Fig. 3. A. A. Czirok, R.N. Mantegna, S. Havlin, H.E. Stanley, Phys. Rev. E52 (1995), 450, fig. 6.

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Zipf law

- Inverse implication not valid!
  (G. Troll, P. Garden, Phys. Rev. E57 (1998), 1347)

- Zipf law does not lead to long range depedence in system,

- However:

$$0 \leq \xi_{shuff} < \xi_{real} \Rightarrow$$ Zipf law indicates correlations

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Correspondence: time series – texts

$x_1, x_2, \ldots, x_N$ – discrete time series

$\Delta x_K = x_{K+1} - x_K$

Δx_j -> $k=2s+1$ – „spin-like" states (s=0, ½, 1, 3/2, …)

$\Delta x_1, \Delta x_2, \ldots, \Delta x_N$ – spin-like 1dim chain (symbolic text)

- k=2 character alphabet (s=1/2)

$$\Delta x_K > 0 \Rightarrow \Delta x_K \rightarrow u \uparrow$$

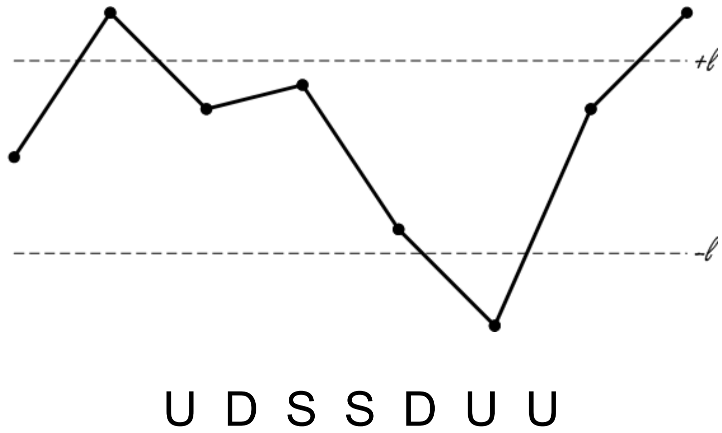
$$\Delta x_K < 0 \Rightarrow \Delta x_K \rightarrow d \downarrow$$

Zipf law

- k=3 character alphabet (s=1)

e.g:

$$|r| \le l \;\Rightarrow\; S$$
$$r > l \;\Rightarrow\; U$$
$$r < -l \;\Rightarrow\; D$$



- k=4 characters: $u\uparrow \;\; U\uparrow \;\; d\downarrow \;\; D\downarrow \qquad \left(s = 3/2\right)$
- k=5 characters: $u\uparrow \;\; U\uparrow \;\; s\updownarrow \;\; d\downarrow \;\; D\downarrow \qquad \left(s = 2\right)$

In all cases TS is devided into „words” of length m
One obtains so called (m,k) Zipf scheme
Usually perdormed for subseries of given length
Every one session by session (local Zipf)

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Predictions for future behavior of T.S.

- One step ahead:
- Assume m-words, k = 2 (u,d)

$$W_{(L)} = word = \left(\underbrace{c_1, c_2, \ldots, c_{m-1}}_{given} \mid \underbrace{l}_{predicted} \right) \qquad l = (u, d)$$

let $$p(l) = probability(conditional) = p(c_1, \ldots, c_{m-1} \mid l)$$

$$p(u) + p(d) = 1 \qquad (1)$$

$$\left. \begin{array}{l} p(u) \sim R_{W(u)}^{-\xi} \\ p(d) \sim R_{W(d)}^{-\xi} \end{array} \right\} \Rightarrow \quad \frac{p(u)}{p(d)} = \left(\frac{R_{W(u)}}{R_{W(d)}} \right)^{-\xi} \qquad (2)$$

(1)+(2) => solution (time dependent for given window)

- Results: M. Ausloos, Ph. Bronlet, Physica A 324 (2003) 30

  Excellent for: NASDAQ

  Very good for: DAX, DJIA, FTSE,

  Poor for: Asian Markets (HANG, SENG, NIKKEI)

Application for WIG20 Futures

- WIG 20 data from 20-12-1999 to 25-05-2010 divided into two parts:

  1) 20-12-1999 to 07-05-2008 – parameters selection

  2) 08-05-2008 to 25-05-2010 – strategy tests
- Words lenght from m = 4 to 6
- Two letters (k=2) (u, d)
- Windows lenght from 200 to 1 000 transaction days

WIG20 close price

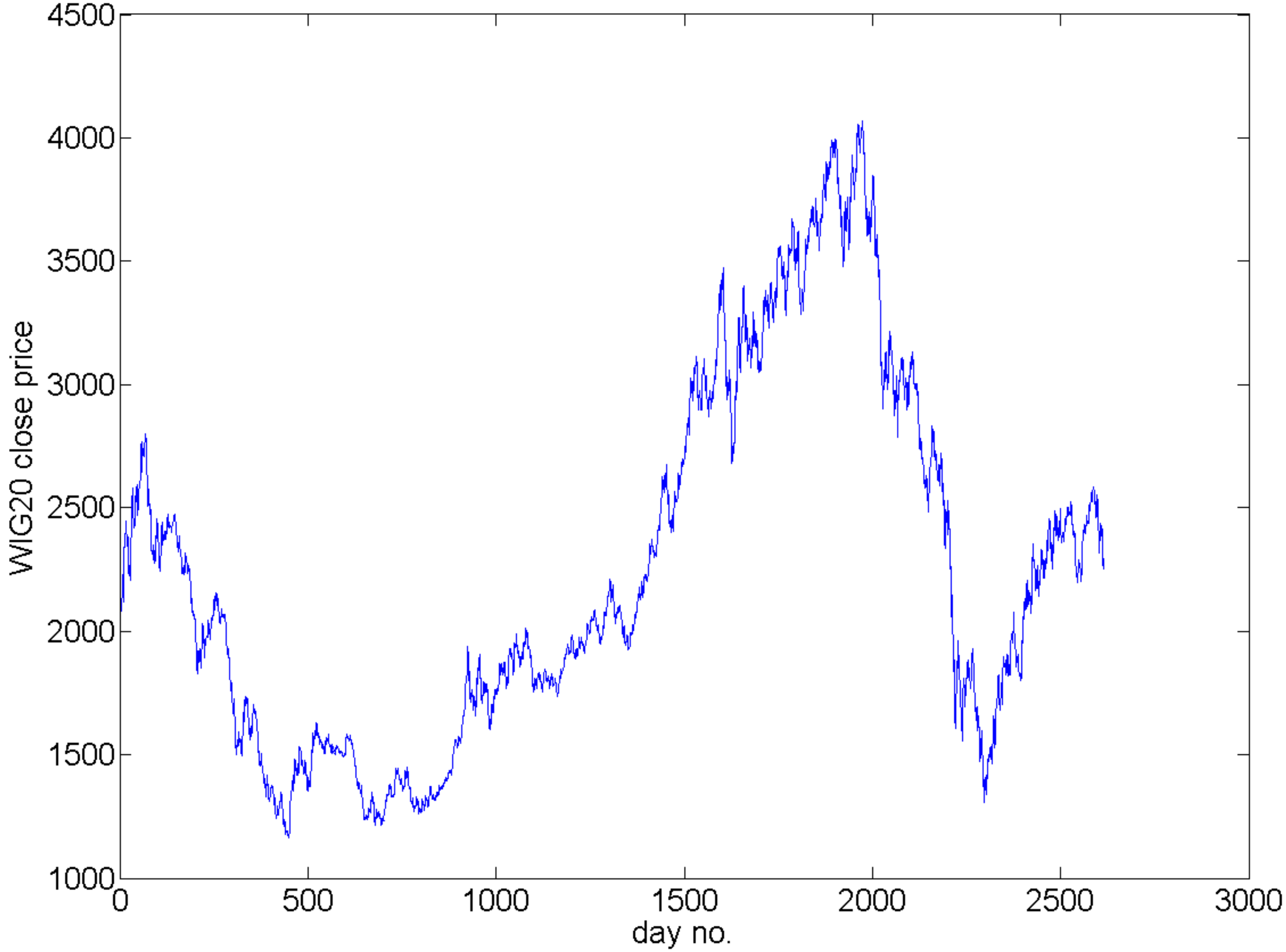

Fig. 4. WIG20 close price

Artificial „day-light” integrated WIG20

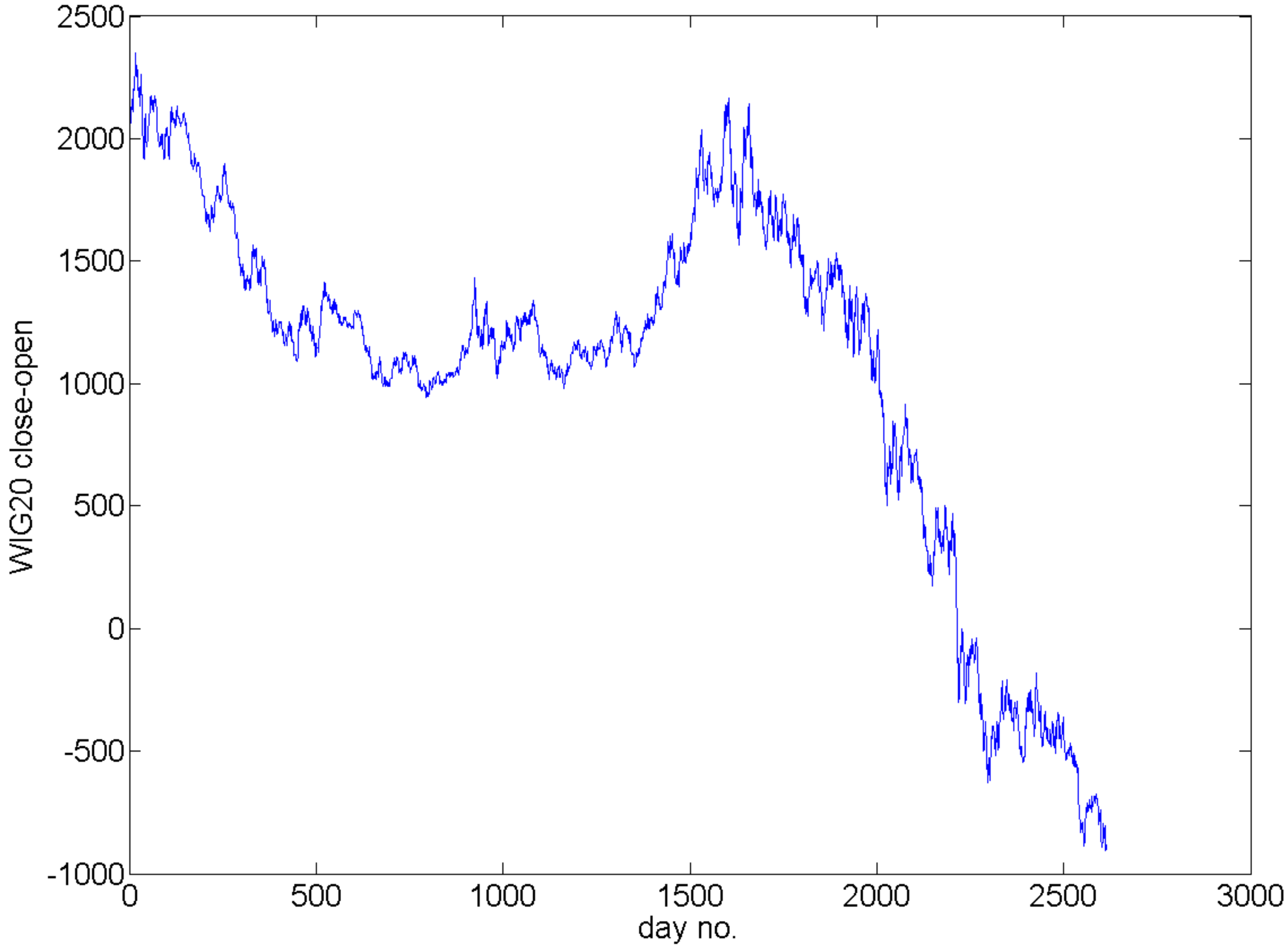

Fig. 5. Artificial „day-light” (close-open) integrated WIG20

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What is „Futures”?

- Futures is standardized contract.
- One party buys contract (long position) and second party sells contract (short position).
- Futures is symmetrical derivative instrument.
- The underlying asset can be:

- Commodities
- Currencies
- Securities
- Stock indexes
- Interest rates
- Others

Payment function

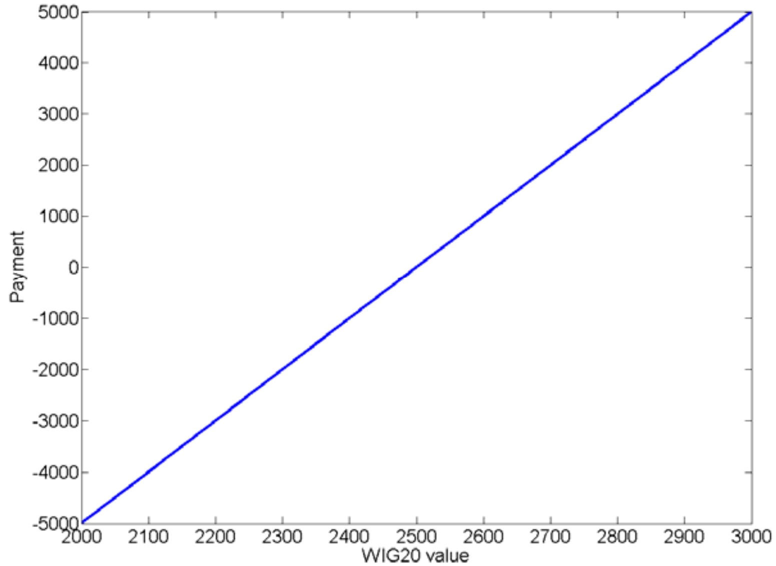


Fig. 6a. Payment function for long position

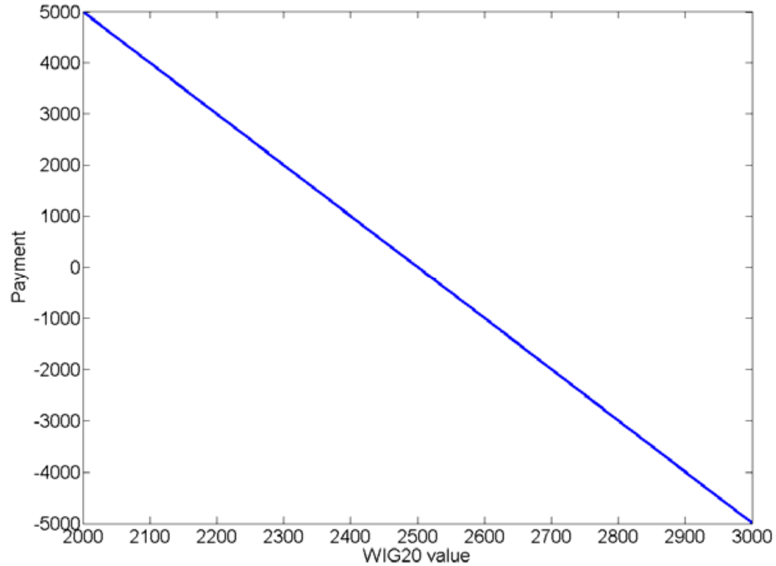


Fig. 6b. Payment function for short position

Assume, that open price is 2 500. Above is presented profit (loss) from investment in WIG20 futures in WIG20 index value function for long (6a) and short (6b) position.

What is „leverage”?

Example:

WIG20 value is 2 500 points and margin is 10%

Investor A buy stocks for 25 000 PLN

Investor B take long position in one WIG20 Futures (margin 2 500 PLN)

WIG20 rise to 2 600 points

Investor A portfolio value is 26 000 PLN (return 4%=(26000–25000)/25000)

Investor B portfolio value is 3 500 PLN (return 40%=(3 500 – 2 500/2 500)

Zipf exponent

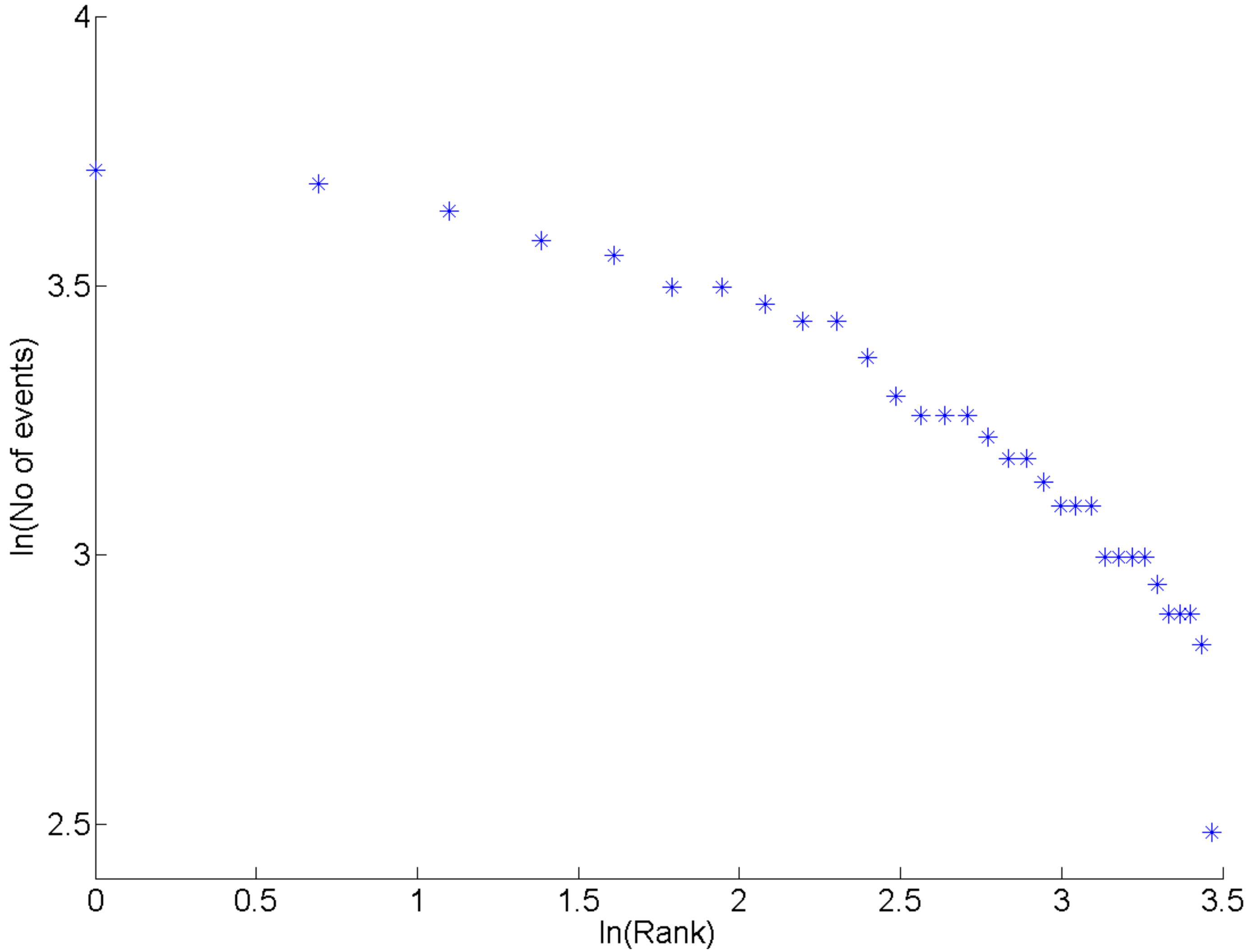


Fig. 7. Real signal (m=5, w=800)

Zipf exponent

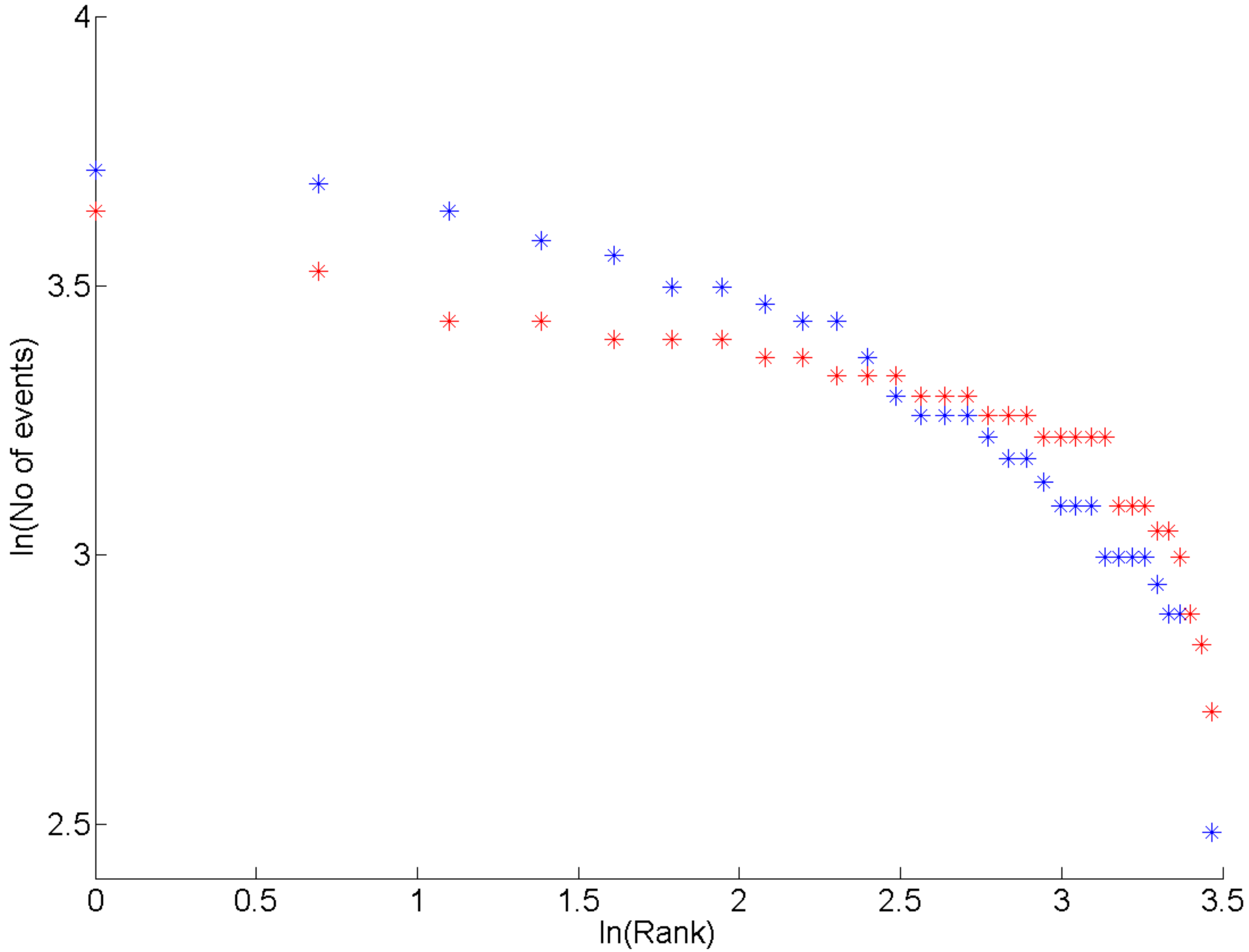


Fig. 8. Real and shuffled signal (m=5, w=800)

Zipf exponent

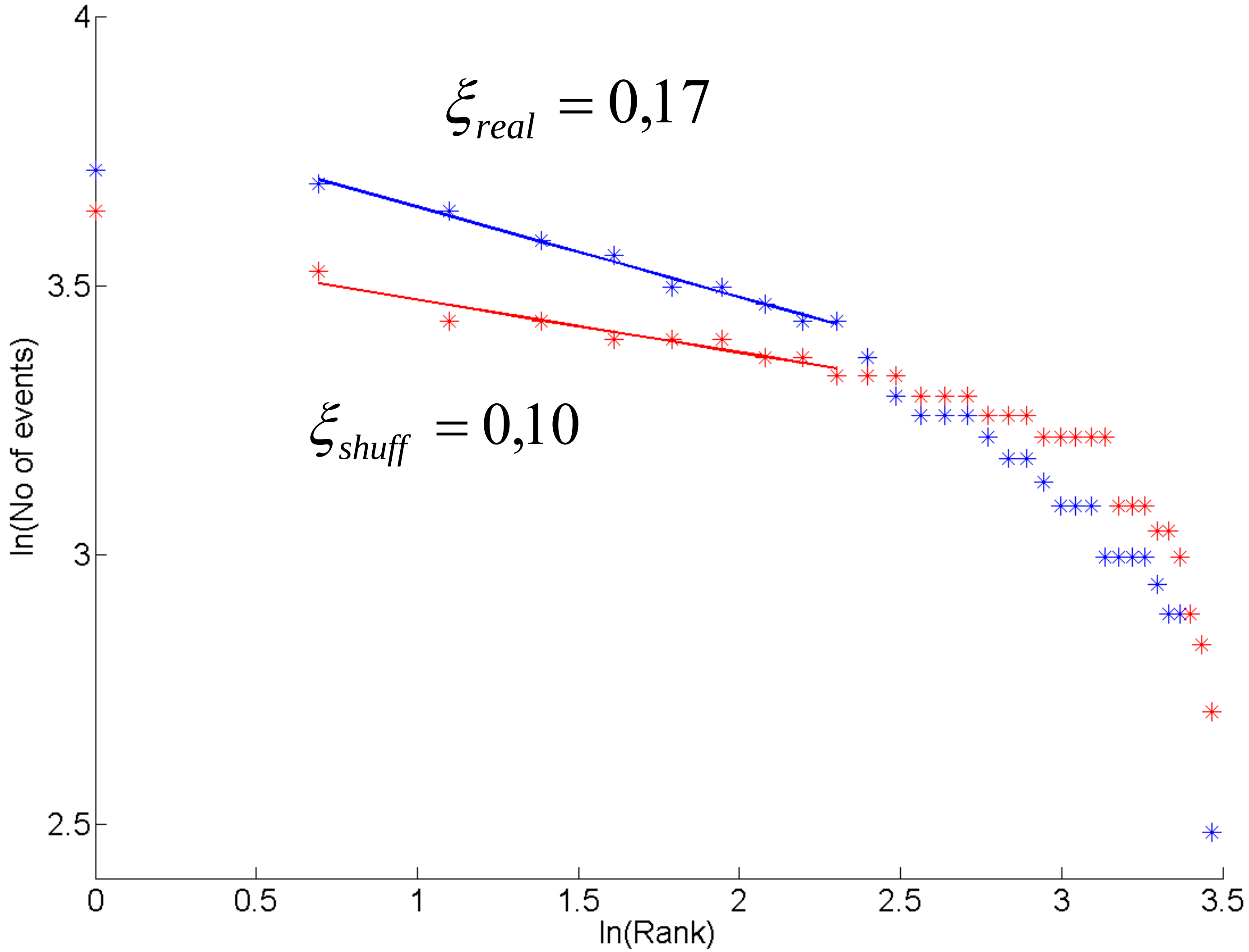


Fig. 9. Real and shuffled signal (m=5, w=800)

Zipf exponent

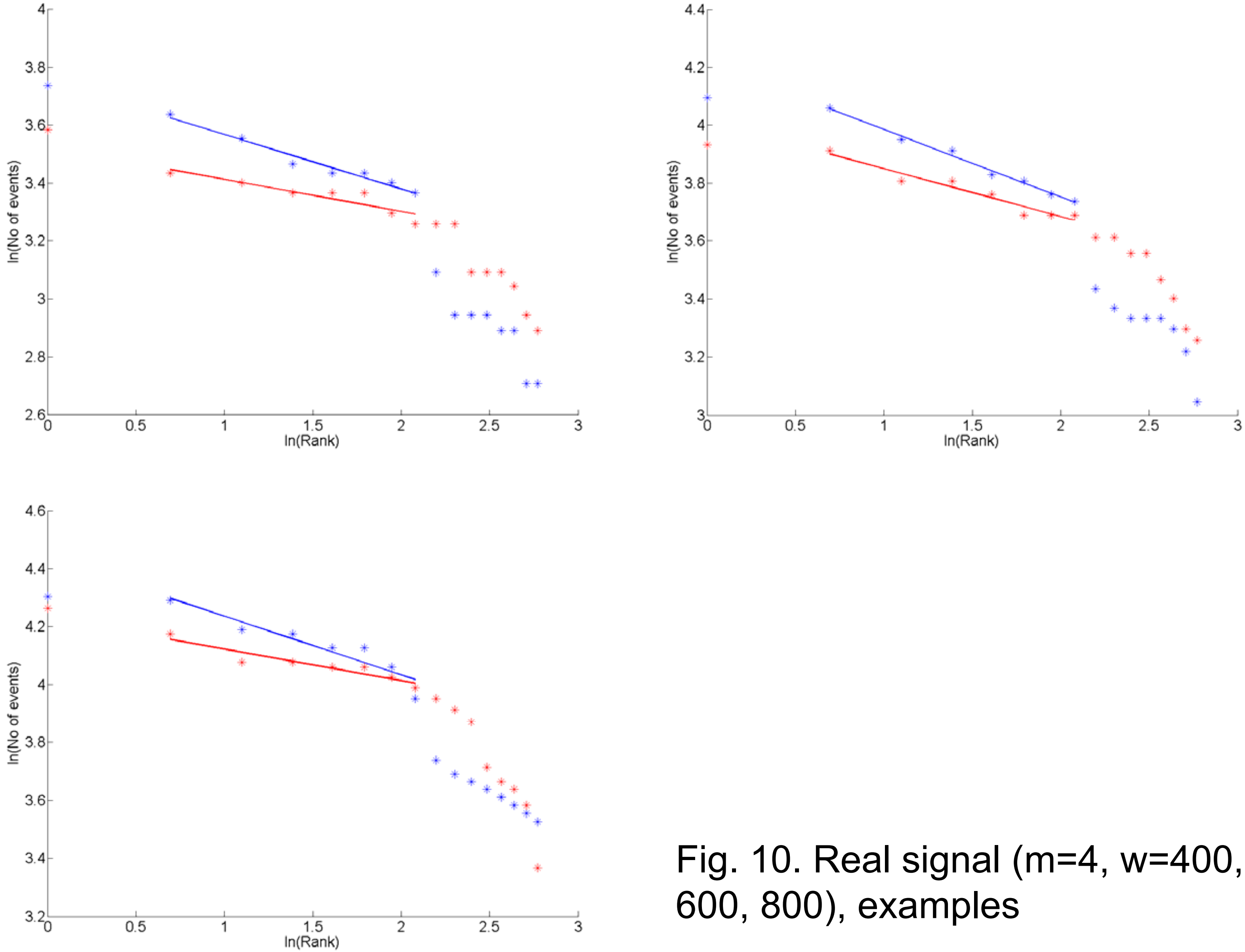
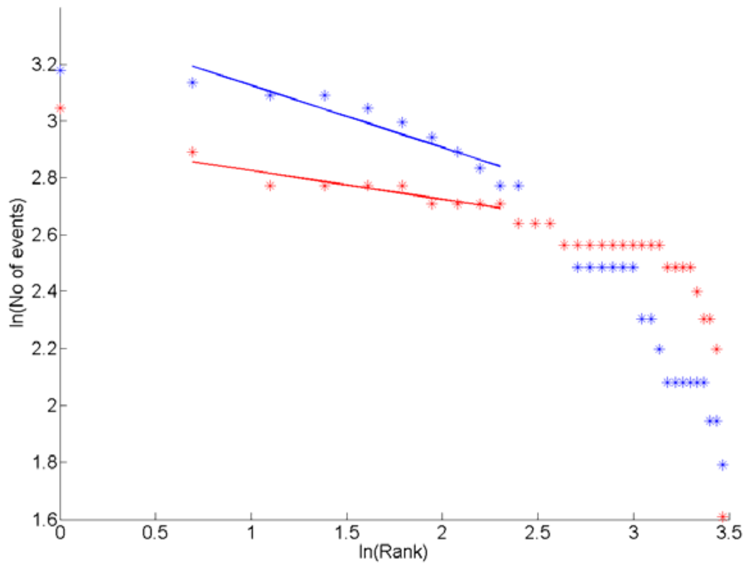
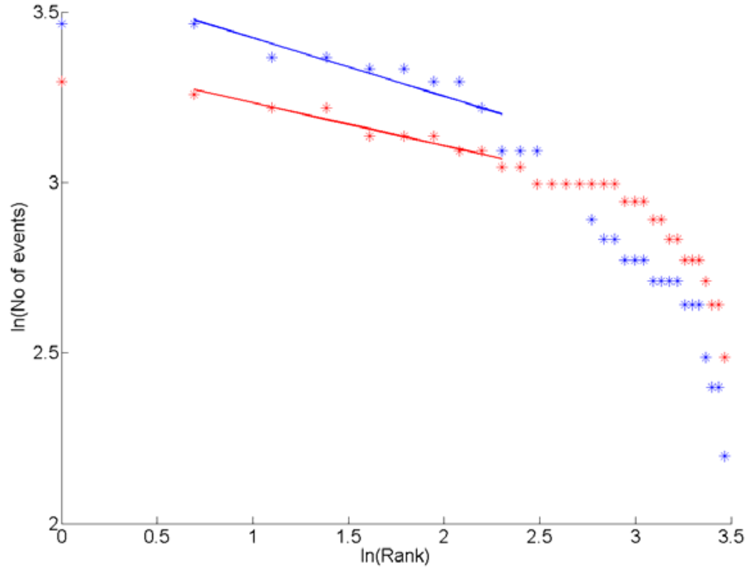


Fig. 10. Real signal (m=4, w=400, 600, 800), examples

Zipf exponent





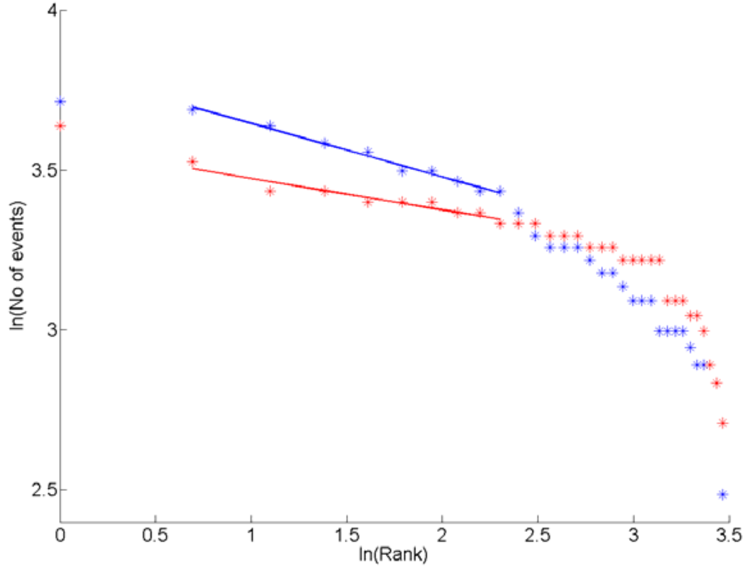
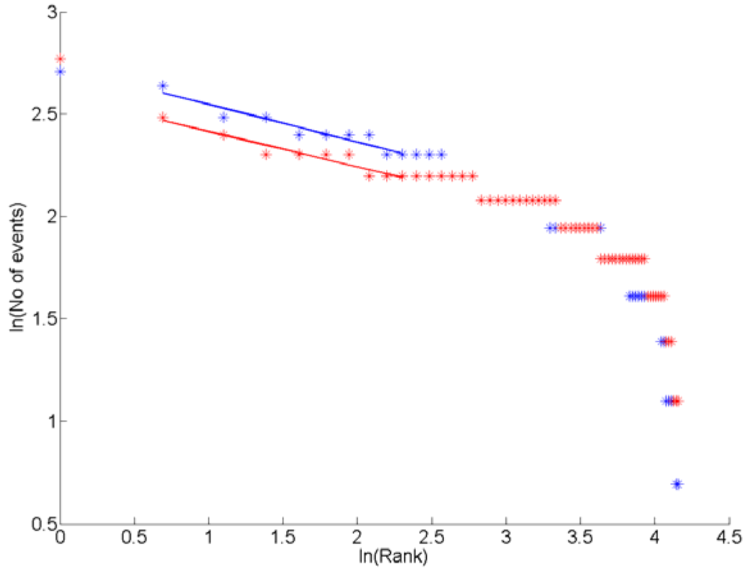
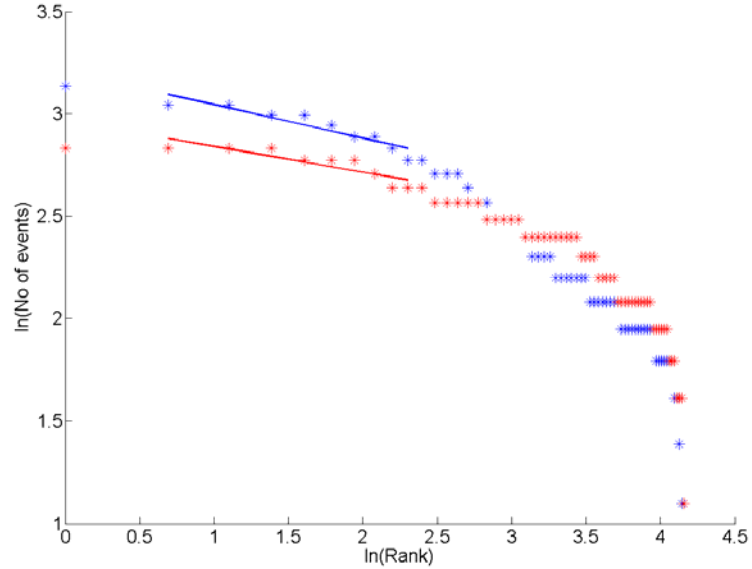


Fig. 11. Real signal (m=5, w=400, 600, 800), examples

Zipf exponent





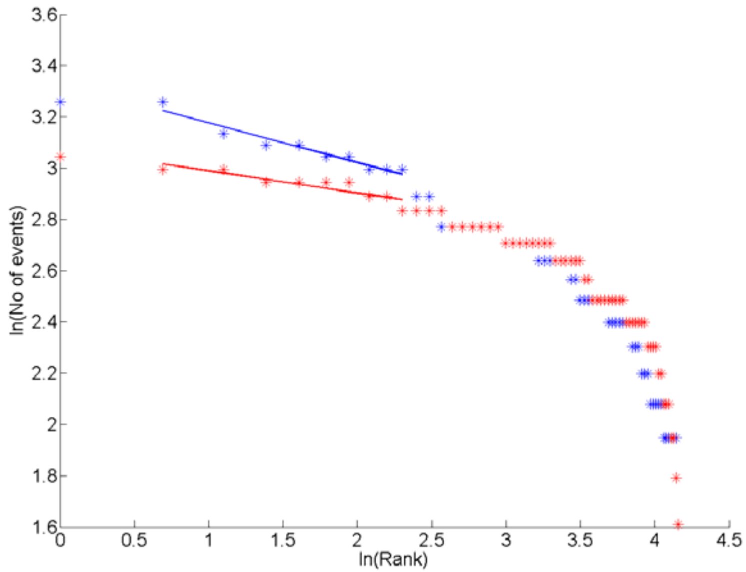


Fig. 12. Real signal (m=6, w=400, 600, 800), examples

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Zipf exponent

Word length (m)	Window length (w)	Zipf exponent for real data	Zipf exponent for shuffled data
4	200	0,2685	0,2397
4	400	0,1881	0,1109
4	600	0,2328	0,1654
4	800	0,2023	0,1089
4	1000	0,1445	0,0897
5	200	0,2776	0,2388
5	400	0,2189	0,1013
5	600	0,1720	0,1265
5	800	0,1671	0,0983
5	1000	0,1804	0,1120
6	200	0,3168	0,1035
6	400	0,1836	0,1728
6	600	0,1635	0,1265
6	800	0,1544	0,0867
6	1000	0,1258	0,1142

Table 1. Zipf exponent for real and shuffled data

Window length (m=4)

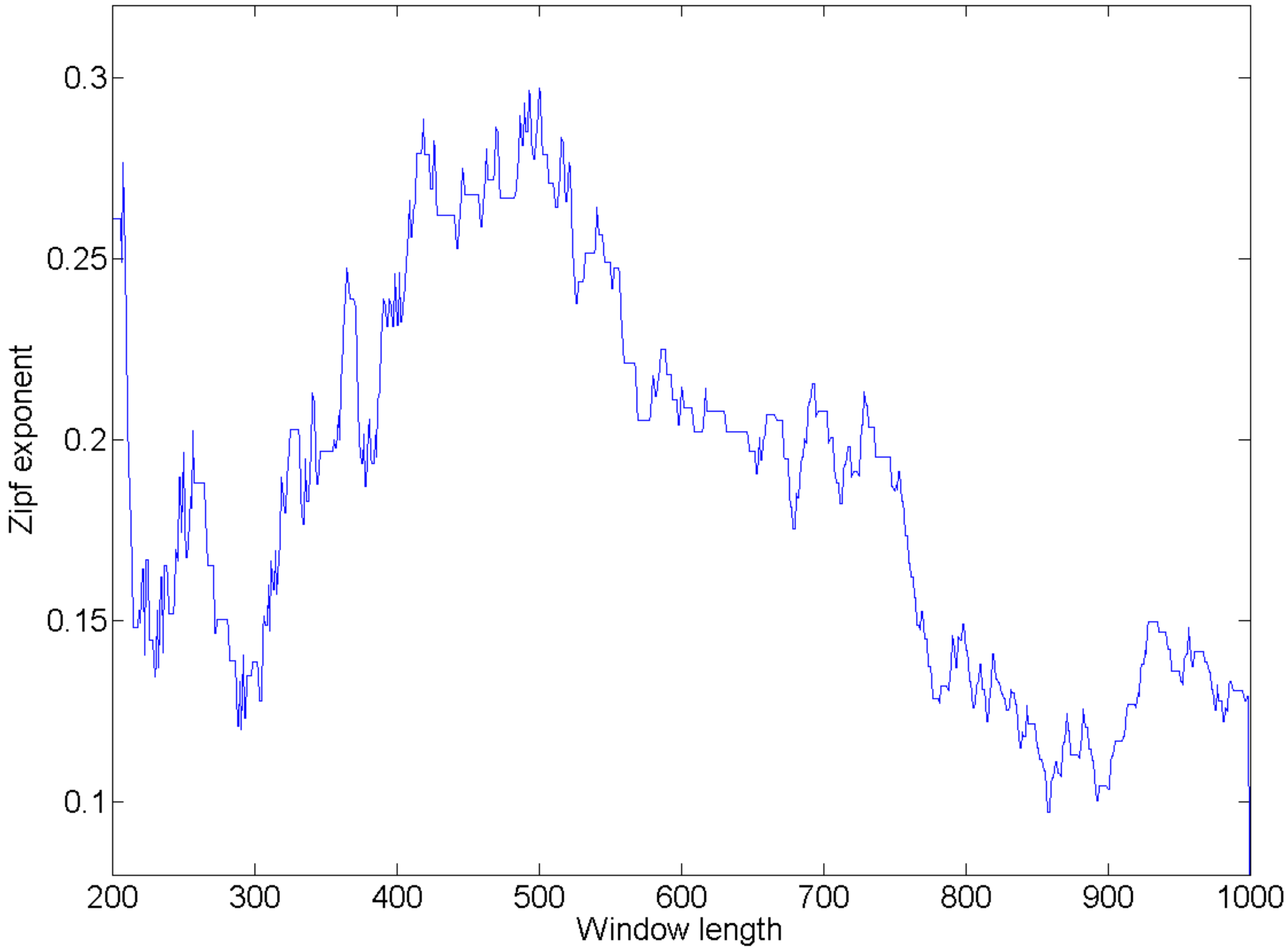

Fig. 13. Zipf exponent vs window length for chosen period (since 18-12-2000)

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Window length (m=5)

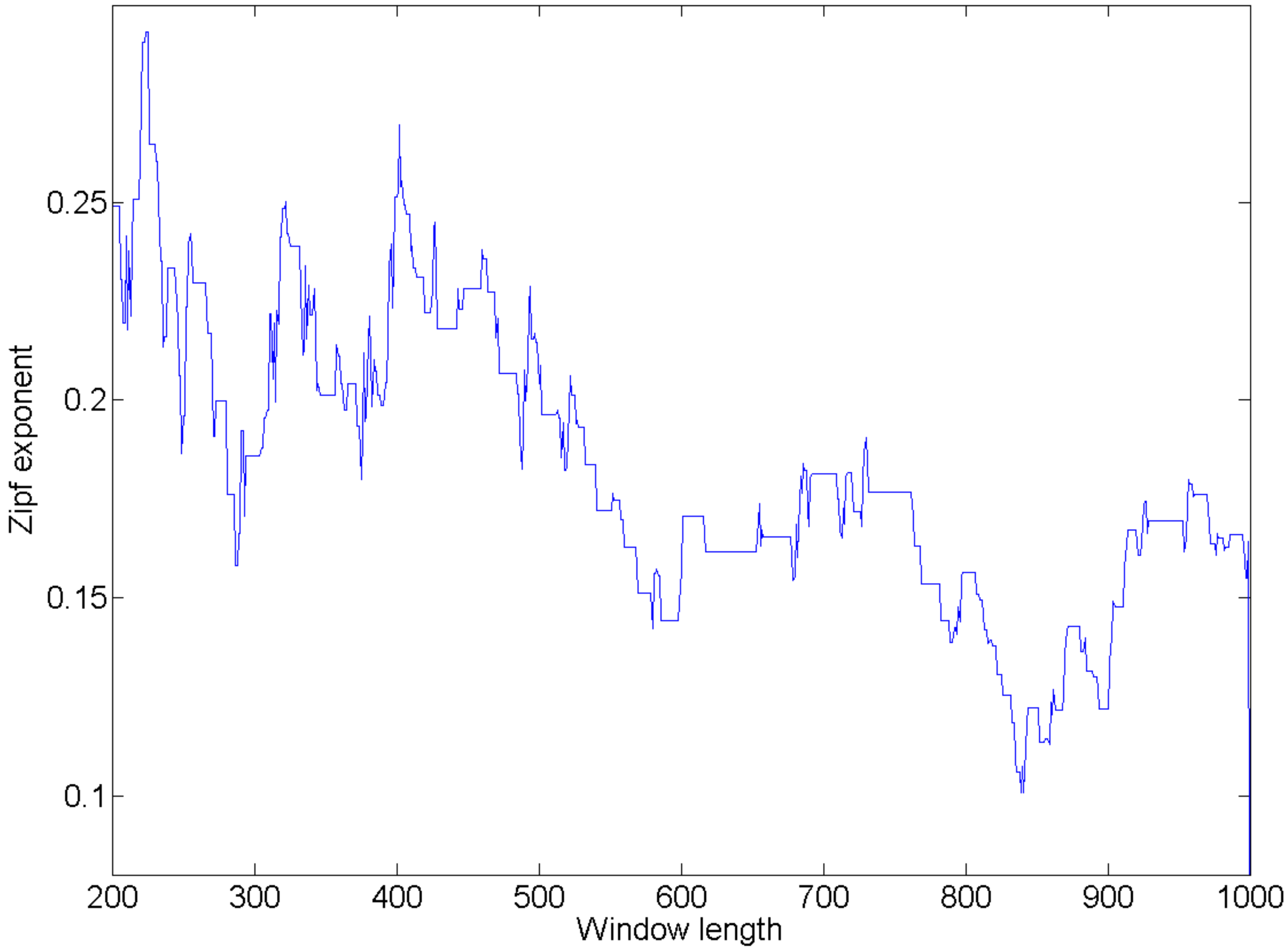

Fig. 14. Zipf exponent vs window length for chosen period (since 18-12-2000)

Window length (m=6)

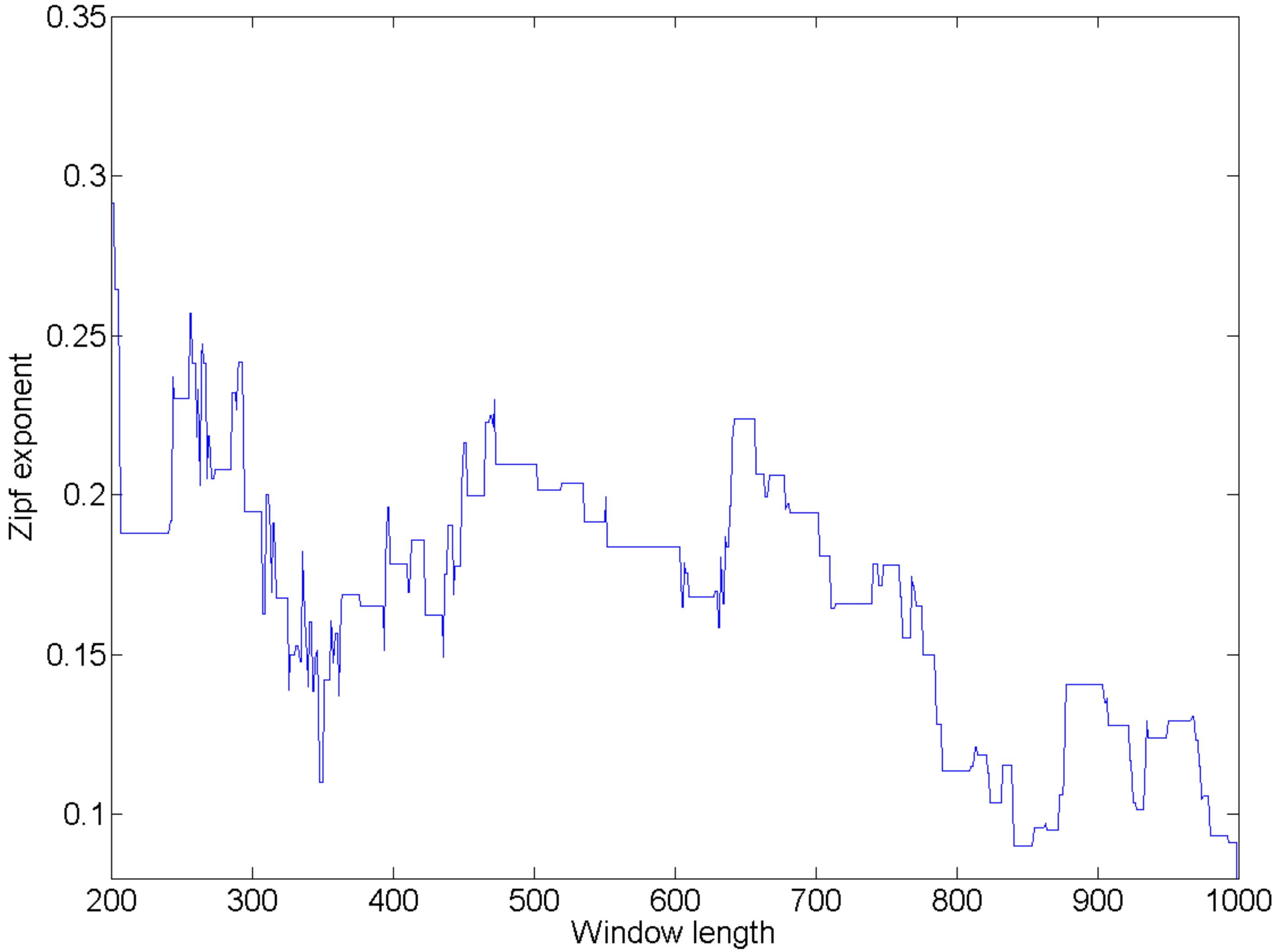

Fig. 15. Zipf exponent vs window length for chosen period (since 01-03-2001)

Strategy results

Strategy „sell and hold” +7 430

Strategy „buy and hold” -7 430

Accuracy

	word lenght		
window length	m=4	m=5	m=6
400	53,0%	53,0%	55,4%
500	53,6%	54,6%	57,0%
600	53,0%	54,4%	55,2%
700	53,0%	54,2%	55,2%
800	53,6%	53,8%	54,8%

Table 2. Accuracy

Profitability (PLN)

	word lenght		
window length	m=4	m=5	m=6
400	4 570	7 630	13 310
500	6 230	9 030	19 230
600	6 070	-190	16 010
700	3 570	5 370	20 130
800	8 390	1 690	15 490

Table 3. Profitability

Strategy results

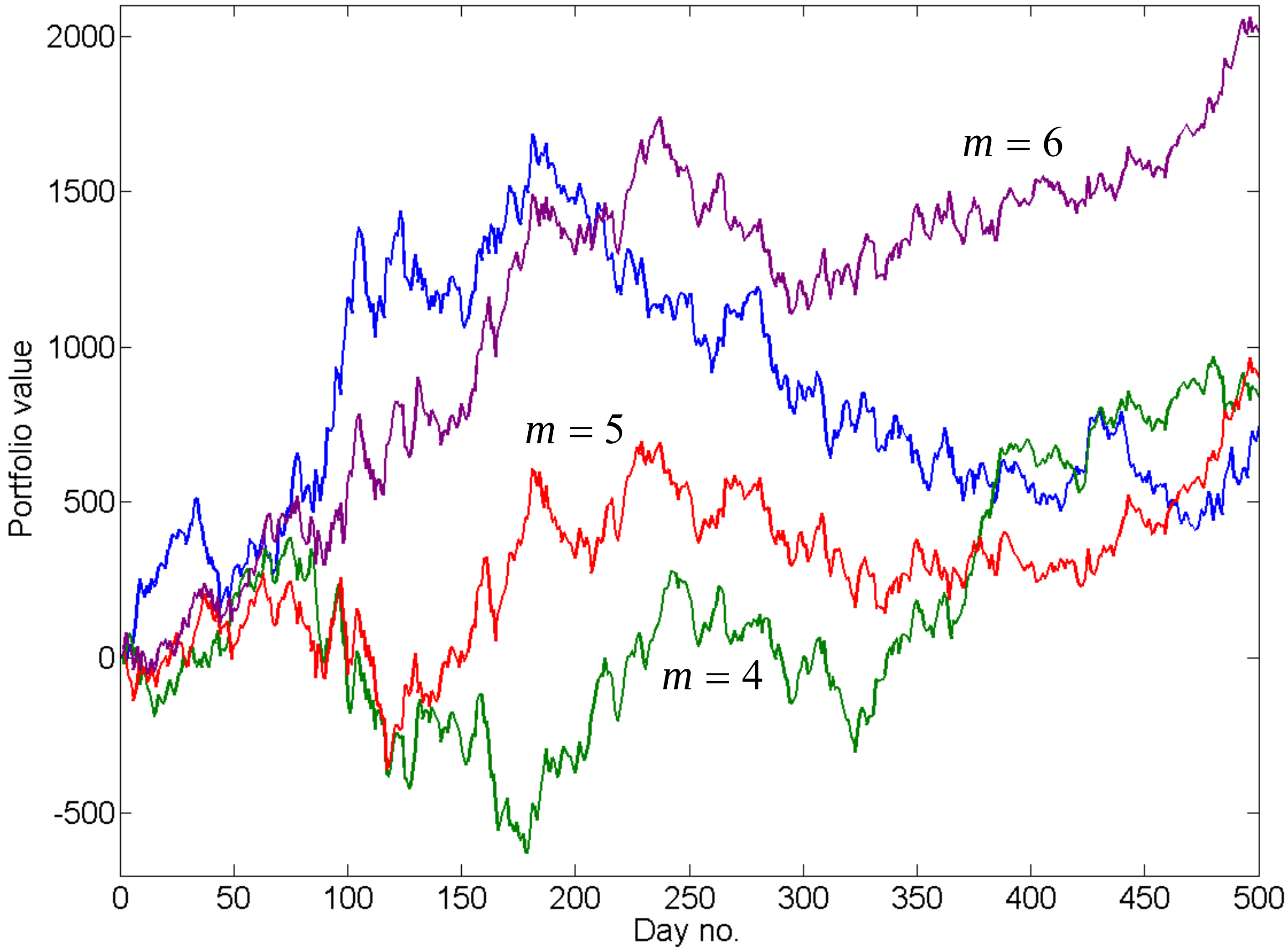

Fig. 16. Strategy results from 08-05-2008 to 25-05-2010

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Conclusions and Perspectives

- Zpif law can be a useful basis for investment strategy for futures
- Zipf strategy gives excellent results independently on current trend (!)
- Long memory in financial data is evident and significantly longer than usually stated (e.g. 4 days).
- Strategy can be applied to other financial, FOREX and commodity market data
- Possibility to make automatic numerical application