

## **Company size and spatial localization – game theory and experimental approach**

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**Abstract:** This paper discusses how the relaxation of the full market saturation assumption with different size companies and under-supplied market influences equilibria in one- and two-dimensional localization games. Game-theory solutions were examined with experiments on localization decisions depending on size of companies. Experimental approach confirms this relationship exists and highlights the inverse relation between companies' size and their mutual distance in equilibrium. Relaxation of assumptions allows the Principle of Minimum and Maximum Differentiation to be borderline cases in localization models. Big companies choose central location (Principle of Minimum Product Differentiation), when small enterprises sprawl to avoid competition (Principle of Maximum Product Differentiation)

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## Introduction

Evolution can be observed in the spatial location theory, which consists in dimensions being added to the firm's space of choice (competition in one-dimensional and two-dimensional space, product characteristics, price), while the number of uncontrollable environmental variables is increasing (consumer distribution, costs of transportation). The objective is to have more application possibilities for location theories, however, there is a risk that unambiguous and interpretable results will not be obtained. In the basic linear city model developed by Hotelling (1929), the assumption is that identical firms offer a homogenous product in a city located along a line of fixed length, and the population density is uniformly distributed. Firms compete with location and price. This is two-stage model where firms choose locations in the first and set prices in the second stage. In this model, a Pareto-effective equilibrium is achieved when the firms are dispersed to the maximum (rule of maximum differentiation). This location maximizes profits in the long-term. Such situation, however, is not a Nash equilibrium, as the companies tend to unilaterally change their location and move towards the centre to maximize short-term gains, which may lead to a situation which is an exact opposite of the original assumption: the firms will choose the same location in the midpoint. As a result of the absence of a stable equilibrium, it is impossible to find an explicit solution to the model (Gupta, 2004). The relocation of the Hotelling model to the space which describes product characteristics became the basis for the application of the principle of minimum differentiation. According to this principle, firms become closer to each other – they offer as similar products as possible. D'Aspremont et al. (1979) proved that with slight variations in the assumptions regarding preferences, the Hotelling model may serve as a basis for the principle of maximum differentiation, which provides that firms move away from one another by manufacturing products with extremely different characteristics.

Salop (1979) suggested an alternative approach, describing a circular city model with uniform consumer distribution. This approach is characterized with market symmetry at any point, in contrast to the Hotelling model, which definitely simplifies the equilibrium analysis for more than two firms. This theory is also more realistic,

reinforcing the application potential of this model. Salop's publication initiated the process of development of the space of analyzed models in order to evolve the spatial competition theory towards more realistic models, and therefore to broaden real-life application possibilities.

The next stage of spatial development in location models was to study location equilibria on a plane, for a city model located over an infinite number of perpendicular lines (Braid, 1986). This theory provides very interesting conclusions regarding equilibria. When compared to the theories developed by Hotelling and Salop, there is a clear analogy between the obtained results. The conclusion from Braid's publication (1986) is as follows: irrespective of the market structure, an increase in the number of direct competitors with a constant population size will result in a fiercer price competition between firms in order to avoid a decline in demand for their product. This will lead to a decline in profits, in proportion to the increase in the number of competitors. The unrealistic assumption of a city located over an infinite number of straight lines makes it impossible to implement this theory in real life, where cities occupy a definite and limited area. In consequence, the Braid's model has remained a theoretical one. Veendorp and Majeed (1995) presented a model of a rectangular city with a two-dimensional uniform consumer distribution. The authors calculated location equilibria for symmetrical location of two firms relative to the centre of the analyzed area. To maximize profits, firms choose partial dispersion, picking locations in the midpoints of opposite sides of the rectangle.

However, all presented models are based on the assumption that the market demand is fully served by the market players, which determines the obtained equilibria. It means that the manufacturing capacities of the firms are strong enough to manufacture a sufficient number of products, and at the same time the price for the products is low enough to convince all consumers to buy them. In reality, this condition can be considered to be met for so-called basic goods (e.g. food). In many cases, however, the number of companies which supply a given product at an acceptable price is limited (e.g. telecommunications services, especially in the

countryside, electricity, gas, petrol etc.). Therefore it is essential to analyze the case where there is no or there is just one supplier of a given product in a market segment. This problem is discussed in this paper. In the first part, location equilibria are determined, which are Pareto-effective Nash equilibria, for a city model through a line of fixed length (Hotelling, 1929) and for a square city model by Veendrop and Majeed, depending on firm size. Firms compete against each other by picking optimum locations, and product prices are the exogenous variable in the model. In the second part of the paper, a discrete square city model is formulated. The calculated theoretical equilibria are then confronted against real-life choices made by people. To this end, an experiment was carried out. The use of experimental economics made it possible to analyze the decision process for real entities and to establish the model equilibria empirically. This approach is particularly useful in the situation where theoretical equilibria: Pareto-effective equilibria and Nash equilibria are divergent, i.e. the analytical solution provides no clear answer to the problem of spatial locations that would be picked by firms in real-life situations, which is the case here.

The use of experimental economics in the analysis of location models is not limited to the verification of the correctness of the model. The level of complexity of location models is so high that it is unlikely that solutions could be found by way of analysis. The analysis of the results of ad hoc location experiments makes it possible to find hidden regularities in location decisions by way of deduction. To meet the demands of the study, the LabSEE software package used to carry out and to visualize economic experiments was further developed.

### **1. Spatial Location Model through a Line of Fixed Length with Variable Firm Sizes**

Let's discuss the Hotelling's Location Model (1929), without the assumption about firm sizes, which satisfies the total market demand. The price of offered goods will be the endogenous variable in our model. The space of choice is a line of

fixed length . Let  $r$  ( $r \in [0, \infty)^1$ ) mean the range of the firm, i.e. the maximum distance that the consumers would be willing to travel to buy the product from the firm. Let  $a$  mean the distance from the beginning of the section to firm A, and  $b$  mean the distance from the beginning of the section to firm B. To simplify the calculations, let's assume that  $a \leq b$ , which does not affect the generality of the model.<sup>2</sup>

In a monopolistic market, the equilibrium demand for the product of A, depending on its size, is  $D_A = 2r$  for  $r \in [0, 1/2)$  and  $D_A = 1$  for  $r \geq 1/2$ , as the range would then be the whole city. A will be located in point  $a$ , where:

$$\begin{cases} a \in [r, 1-r] & dla \quad r \in [0, 1/2) \\ a \in [1-r, r] & dla \quad r \in [1/2, 1) \\ a \in [0, 1] & dla \quad r \geq 1 \end{cases}$$

The case where only one firm is present on the market produces quite intuitive results. For a small firm, which is unable to satisfy the entire market demand, there are many optimum options. However, we know that this firm will not choose an outermost location, at the end of the analyzed area, as the range of the firm would exceed the city area. With the increase of the range of the firm, the distance to the boundaries of the area will increase.  $r = 1/2$  is the extreme value, for which there is a clear equilibrium in the centre. At  $r \geq 1/2$ , firms will fully satisfy the market demand. The longer the firm range is, the farther the firm is able to move away from the centre of the city. When  $r \geq 1$ , the firm meets the total market demand irrespective of its location, which is an assumption of the Hotelling's model (1929).

For two firms, A and B, with identical  $r$ , the demand for their products will be respectively:

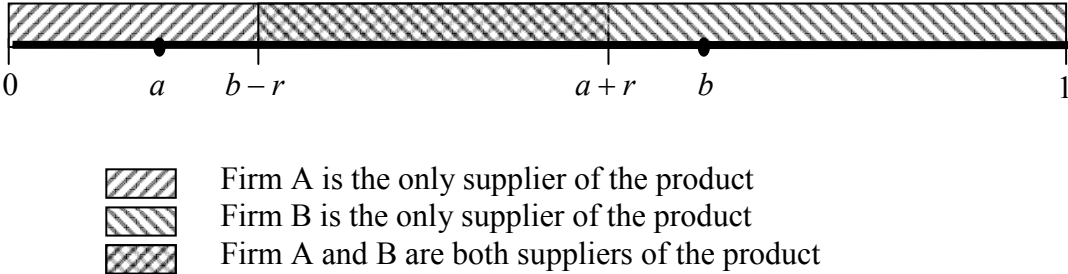
$$\begin{cases} \begin{cases} D_A = 2r \\ D_B = 2r \end{cases} & dla \quad r \in [0, 1/4) \\ \begin{cases} D_A = (b-r) + \frac{a+r-(b-r)}{2} \\ D_B = (a+r) + \frac{a+r-(b-r)}{2} \end{cases} & dla \quad r \in [1/4, 1/2) \\ \begin{cases} D_A = 1/2 \\ D_B = 1/2 \end{cases} & dla \quad r \geq 1/2 \end{cases}$$

<sup>1</sup> In a basic Hotelling's model:  $r \geq 1$

<sup>2</sup> This condition means that A is located to the left from B or they are located in the same spot.

For  $r \in [0, 1/4)$ , the firms avoid competition and divide the market into two equal parts. With a range of more than  $1/4$ , the firms are able to obtain demand in excess of  $1/2$ . It means that the firms will be forced to compete against each other. The first part of the demand equation shows demand in the market section where the analyzed firm is the only supplier of the product. The second part of the equation applies to a duopoly market, where each of the firms supplies half of the market. Figure 1 represents the division of the market between the two firms in the analyzed situation.

**Figure 1: Division of the market between the two firms**



With the increase in the firm size, the distance to the boundaries of the analyzed area will increase, and the competitive activity will cover a larger part of the market. Large firms ( $r \geq 1/2$ ) will compete across the entire market. To maximize the demand for their products, they will choose a location in the very centre of the analyzed area ( $a = b = 1/2$ ).

To summarize the above calculated results, depending on the range of the firms present on the market, we obtain:

$$\begin{cases} D_A = D_B = 2r & \text{dla } r \in [0, 1/4) \\ D_A = D_B = 1/2 & \text{dla } r \geq 1/4 \end{cases}$$

Both firms enjoy identical demand due to the symmetry of the market. The location equilibrium ( $a, b$ ) in the analyzed model is as follows:

$$\left\{ \begin{array}{l} a \in [r, 1-3r] \\ b \in [a+2r, 1-r] \end{array} \right. \quad dla \quad r \in [0, 1/4)$$

$$\left\{ \begin{array}{l} a = r \\ b = 1-r \end{array} \right. \quad dla \quad r \in [1/4, 1/2)$$

$$\left\{ \begin{array}{l} a = 1/2 \\ b = 1/2 \end{array} \right. \quad dla \quad r \geq 1/2$$

The obtained results indicate that small firms ( $r \in [0, 1/4)$ ) will disperse to avoid competition, as they are able to divide the market between themselves. Medium-sized firms ( $r \in [1/4, 1/2)$ ) will compete over some part of the market. However, they will remain local monopolists and will maximize the size of their respective markets. Large firms ( $r \geq 1/2$ ) will be located in the middle of the segment in order to obtain a dominant market position, since there will be duopolistic conditions all over the area.

Therefore it is clear that when firm sizes are taken into consideration, they affect the location of firms in the state of equilibrium. The Hotelling's Model (1929) is a special case of the analyzed model where  $r \geq 1$ , which means that firms compete over the entire market irrespective of their location.

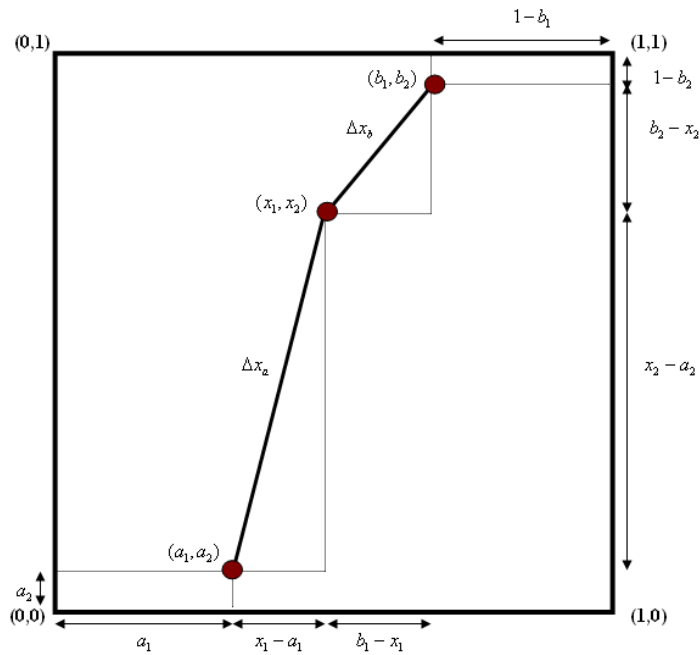
## 2. Spatial Location Model for a Two-Dimensional Space with Variable Firm Sizes

The next stage of the study is to describe location equilibria in a square city model developed by Veendorp and Majeed (1995) depending on firm sizes. Let's assume that consumers are uniformly distributed along the sides of a square. Each of the sides equals 1. The price of offered products is the endogenous variable in this model. Let  $r$  ( $r \in [0, \infty)$ )<sup>3</sup> mean the range of the firm, i.e. the maximum distance that the consumers would be willing to travel to buy the product from the firm. If  $A = (a_1, a_2)$  and  $B = (b_1, b_2)$  are the locations of A and B respectively relative to  $(0,0)$ , and  $X = (x_1, x_2)$  is the location of a model consumer relative to the origin of coordinates, then the distance from customer X to firms A and B is:  $\sqrt{(x_1 - a_1)^2 + (x_2 - a_2)^2}$  and  $\sqrt{(x_1 - b_1)^2 + (x_2 - b_2)^2}$ , respectively. To simplify the calculations, let's assume that A is located closer to  $(0,0)$  than B, i.e.

<sup>3</sup> In a basic Hotelling's model:  $r \geq 1$ .

$\sqrt{a_1^2 + a_2^2} < \sqrt{b_1^2 + b_2^2}$  Figure 2 presents the examples of locations of two firms (A and B) and a consumer (X) in the analyzed model.

**Figure 2: Two firms (A and B) and customer (X) in square city model**



Let's start the equilibrium analysis with the monopolistic market, where only a single firm is present. The firm location which maximizes the demand for its product depending on the range is shown in the table below.

Firm range ( $r$ )	Demand ( $D_A$ )	Location ( $a_1, a_2$ )
$[0, 1/2)$	$\pi r^2$	$\forall_{i=1,2} a_i \in [r, 1-r]$
$[1/2, \sqrt{2}/2)$	$\pi r^2 - 8 \int_{1/2}^r \sqrt{r^2 - x^2} dx$	$\forall_{i=1,2} a_i = 1/2$
$[\sqrt{2}/2, \sqrt{2})$	1	$\left\{ \begin{array}{l} a_1 \in \left[ \left[ 1 - \sqrt{r^2 - 1/4}, \sqrt{r^2 - 1/4} \right] \cap [0, 1] \right] \\ a_2 \in \left[ \left[ 1 - \sqrt{r^2 - a_1^2}, \sqrt{r^2 - a_1^2} \right] \cap [0, 1] \right] \end{array} \right.$
$\geq \sqrt{2}$	1	$\forall_{i=1,2} a_i \in [0, 1]$

Small firms ( $r \in [0, 1/2)$ ) are in equilibrium when the entire range is inside the analyzed area. Medium-sized firms ( $r \in [1/2, \sqrt{2}/2)$ ), seeking to minimize the range outside the city borders, will be located in the very centre of the city. Large firms

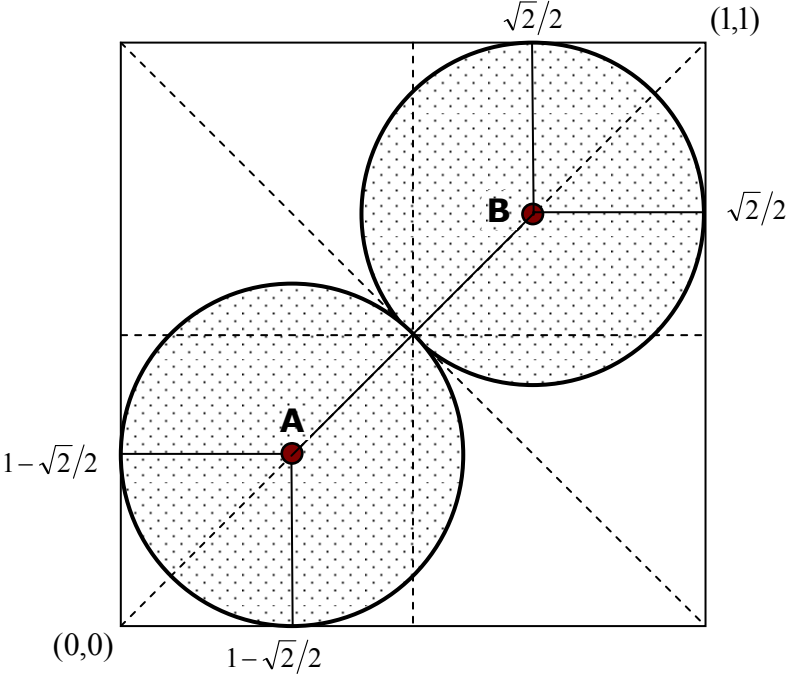


$(r \geq \sqrt{2}/2)$  will have more location possibilities with the increase of their range, since all consumers located in the city will buy from them. The results obtained from this model coincide with the results obtained from the location model for a line of fixed length. This means that in a monopolistic market, the choice of space in the analyzed model does not have a substantial effect on the results.

If there are two firms present on the market, the aspect of competition arises, in order to maximize the area where consumers will choose one product over the other competing product.

Just like in the location model for a line of fixed length, small firms  $(r \in [0, 1 - \sqrt{2}/2])$  will be able to avoid competitors by choosing a location which is distant from the centre of the analyzed area. The critical point is  $r = 1 - \sqrt{2}/2$ , where firms choose locations on the diagonal:  $A = [1 - \sqrt{2}/2, 1 - \sqrt{2}/2]$ ,  $B = [\sqrt{2}/2, \sqrt{2}/2]$  and their entire range will fit inside the analyzed area without the need to compete, which is demonstrated in the picture below.

**Figure 3: Two firms (A and B) as local monopolists**



However, as the range grows  $(r > 1 - \sqrt{2}/2)$ , the firms are forced to compete against each other. They will gradually move along the diagonal of the square

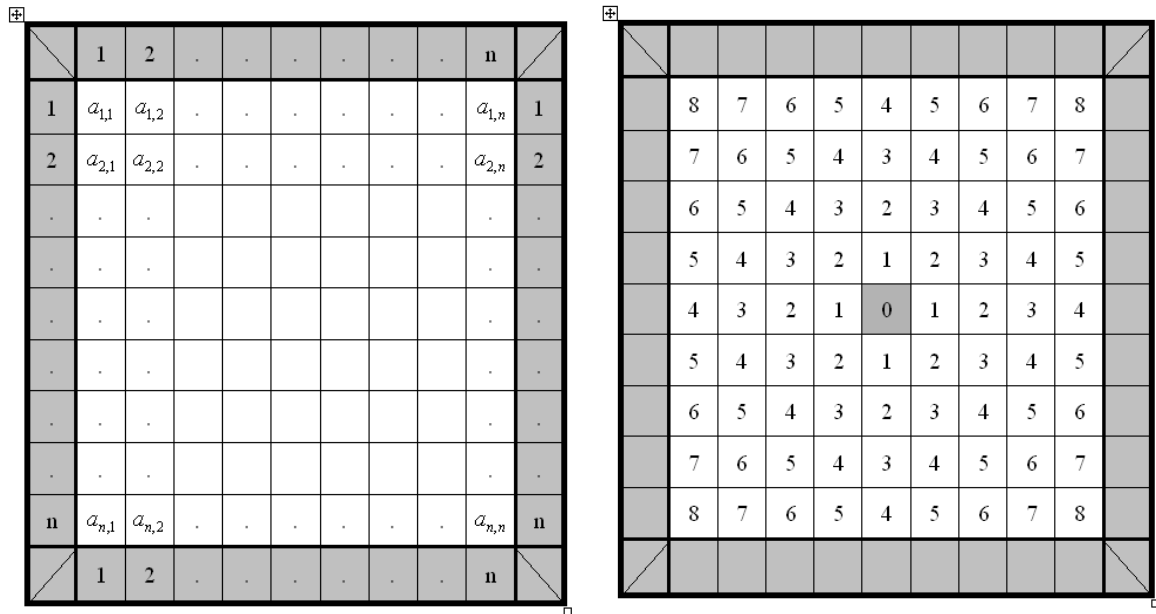
towards the midpoint. Large firms, with the range of  $r \geq \sqrt{2}$  will choose a location in the very centre of the analyzed area.

The equilibria calculated for spatial location models, both for a line of fixed length (one-dimensional) and for a square city (two-dimensional city), indicate that firms can be divided into three groups by size: small firms, medium-sized firms and large firms. Small firms will choose maximal differentiation (peripheral) locations in order to become local monopolists. Medium-sized firms, being forced to compete against each other (medium differentiation), will choose locations closer to the middle of the analyzed area. However, they will aim to maximize the area where they remain local monopolists, therefore the location will not be too close to the centre. Large firms will choose central locations (minimal differentiation) in the very middle of the area, having a range wide enough to satisfy the entire market demand. A duopolistic competition between the two firms will cover the whole market. They will divide the market between themselves in two equal parts.

### **3. Experimental study of location equilibria**

In order to verify whether the calculated theoretical equilibria of the models are actually confirmed in reality, a special platform was created to carry out experiments regarding the choice of spatial locations. The space was described by means of a square grid of  $n$  rows and  $n$  columns, where  $n = 1, 2, \dots, \infty$ . This way, the space comprised  $n^2$  identical squares. Defining the values for each field ( $a_{i,j}$  for  $i, j = 1, 2, \dots, n$ ), we map the population density in the area under study which determines the demand in each subunit. This situation is shown in the figure below (Figure 4a).

**Figure 4. Discussed model: a) Demand in each subunit and b) Consumer distance (point 0) to subsequent fields**



Owing to such an approach to defining the distribution of consumers, all models shown above represent special cases of this problem. For example, if we assume:  $n \rightarrow \infty$ ,  $a_{i,j} = 1/n$  where  $i = 1, j = 1, 2, \dots, n$  and  $a_{i,j} = 0$  in other cases, and the length of the side of each subunit is  $1/n$ , we obtain a city model on a line of fixed length, while for  $n \rightarrow \infty$ ,  $a_{i,j} = 1/n^2$  where  $i, j = 1, 2, \dots, n$  and the length of the side of each subunit is  $1/n$ , we obtain a square city model.

Only one homogeneous product is offered on the market. Each of the consumers can buy one unit at maximum. A transaction will occur when the benefit from the transaction is equal to or exceeds the incurred costs, i.e.

$$\max_{k=1,2,\dots,K} (v - (p_k + t\Delta x_k)) \geq 0$$

where:  $v$  - consumer benefit from the purchase of the product,  $p_k$  - price set by firm  $k$ ,  $t$  - transportation costs,  $\Delta x_k$  - distance from the customer to firm  $k$ ,  $k = 1, 2, \dots, K$ . Of course consumers demand products from the firm for which the costs of purchase are the lowest.

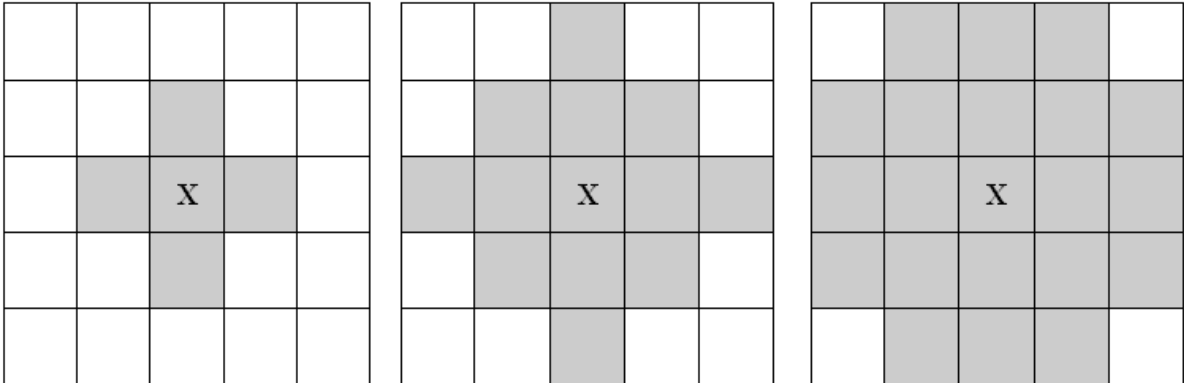
Firms maximize their profit, which is defined as follows:

$$\pi_k = (p_k - c)Q_k, \text{ for } k = 1, 2, \dots, K$$

where:  $\pi_k$  - profits of firm k,  $p_k$  - sale price of the product, c- unit cost of production,  $Q_k$  - demand for a product offered by firm k.

To simplify the calculations, the model assumes that the traffic flows only in the following directions: north, south, east and west, which means that it is impossible to move along the diagonal.<sup>4</sup> Having defined such a measure, the distance from the consumer to subsequent fields is presented in the figure above (Figure 4b). The area under study was divided into 25 subunits, i.e.  $n = 5$ . Uniform distribution of consumers was assumed, i.e.  $a_{i,j} = 1$  where  $i, j = 1, \dots, n$ . Firms sell only one homogenous product at the same, exogenous price  $\bar{p}$ . Therefore they compete against each other, choosing a location to maximize their profits. Three different firm sizes were taken into consideration, in terms of their range: small firms: range = 1, medium-sized firms: range = 3 and large firms: range = 5 which is presented in the figure below.

**Figure 5: A small / medium-sized / large firms located at point X**



The figure above confirms that the wider the range of the firm is, the more customers the firm is able to serve. Profits are proportionate to the demand for the supplied product. It is noteworthy that the analyzed situation is a special case of the generic model presented above. This situation arises from the following assumptions:

- Small firm:  $v - \bar{p} = 1, \bar{p} - c = 1$ ;
- Medium-sized firm:  $v - \bar{p} = 3, \bar{p} - c = 1$ ;

<sup>4</sup>This way, the defined measure distance seems to be quite intuitive, although it can be replaced with any other measure.

- Large firm:  $v - \bar{p} = 5$ ,  $\bar{p} - c = 1$ .

Based on the model presented above, situations where two firms coexist on the market were studied. They were small firms, medium-sized firms and large firms respectively. Nash equilibria and Pareto-effective equilibria were determined for these firms, and then the equilibria were verified by way of experiments.

For small firms there are 36 Nash equilibria, only 20 of which are Pareto-effective, which is presented in the figure below.

**Figure 6: Nash Equilibria (NE) and Pareto-effective Nash Equilibria (NE) in a model with two small firms, where the rows of the matrix represent the location strategies of player X, and the columns of the matrix represent the location strategies of player Y**

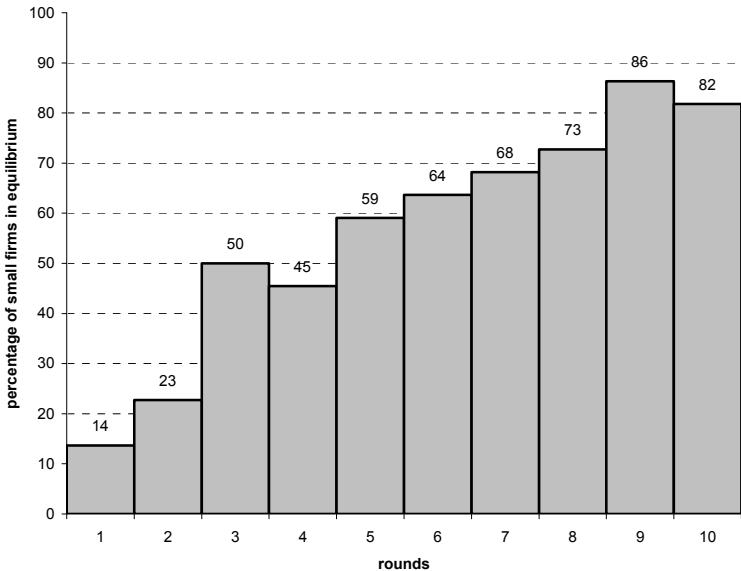
NE		Player Y strategy																									NE	
		1;1	1;2	1;3	1;4	1;5	2;1	2;2	2;3	2;4	2;5	3;1	3;2	3;3	3;4	3;5	4;1	4;2	4;3	4;4	4;5	5;1	5;2	5;3	5;4	5;5		
Player X strategy	1;1																										1;1	
	1;2																											1;2
	1;3																											1;3
	1;4																											1;4
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NE		Player Y strategy																									NE	

As you can see in the figure above, small firms are partly dispersed in equilibrium conditions. They pick locations so as not to compete against each other, thus maximizing their profits. Of course picking a location on the boundary of the area would be irrational, as it would reduce the profit. The situation where a firm is located in the very middle of the discussed area is similar, as it forces the firm to compete against its rival.

In order to verify the theoretical results, an experiment was carried out. There were 44 people in the investigated group. Two people at a time took part in the

experiment. The game consisted of 10 identical rounds. In each round, both players were expected to pick a location for their firm at the same time. The percentage of player pairs in Nash equilibria and Pareto-effective equilibria in each round is presented in the figure below.

**Figure 7: Percentage of small firms in equilibrium in each round**



Round by round, more and more players came to an agreement and eventually 80% of them reached the location equilibrium. The time needed to reach the equilibrium varied depending on the decisions of the players, which were influenced by the predictability of the opponent's moves. The firms which succeeded in reaching the equilibrium remained in a location which guaranteed maximum profits to the very end of the game. However, just under 20% of the players failed to come to an agreement and to reach the equilibrium. The reason behind it was a desire for a central location, which was against reason. Those players remained permanently in the middle of the area, irrespective of the decisions made by their opponent. In consequence, the profits of both firms declined. However, they found the awareness of being strategically located more valuable than earnings.

For medium-sized firms there are only 4 Nash equilibria, and all of them are Pareto-effective, which is presented in the figure below. They are reached for the following firm locations:  $((3,2),(3,4))$ ,  $((2,3),(4,3))$  and locations which are

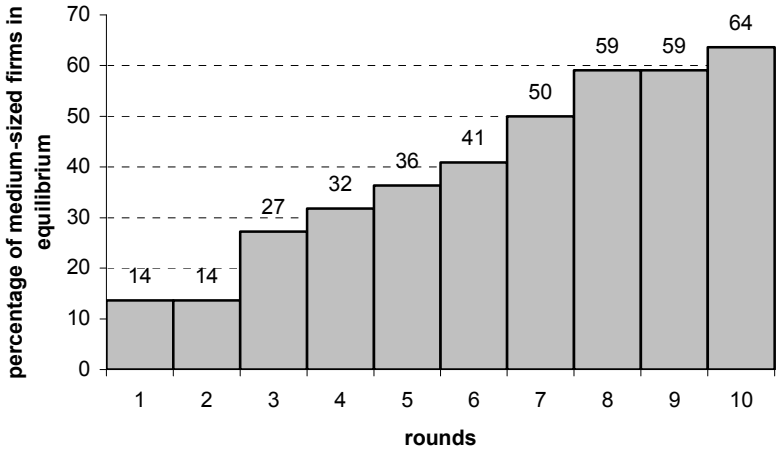
symmetrical to those locations. Therefore we can see a quite intuitive relation: the larger the size of competing firms is, the lower is the dispersion of the competitors.

**Figure 8: Nash Equilibria (NE) and Pareto-effective Nash Equilibria (NE) in a model with two medium-sized firms, where the rows of the matrix represent the location strategies of player X, and the columns of the matrix represent the location strategies of player Y**

NE		Player Y strategy																									NE		
		1;1	1;2	1;3	1;4	1;5	2;1	2;2	2;3	2;4	2;5	3;1	3;2	3;3	3;4	3;5	4;1	4;2	4;3	4;4	4;5	5;1	5;2	5;3	5;4	5;5			
Player X strategy	1;1																											1;1	
	1;2																												1;2
	1;3																												1;3
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NE		Player Y strategy																									NE		

Only 64% firms achieved the location equilibrium in this game, which is definitely less than in the previous game. This was caused by an even greater desire to have a central location than in the case of small firms.

**Figure 9: Percentage of medium-sized firms in equilibrium in each round**



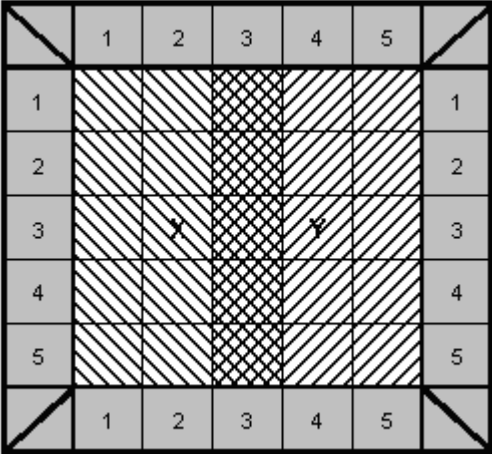
In this situation, as many as 39% people initially located their firms in the middle of the area. In the rounds that followed, the number oscillated between 20-30%, to fall slightly below 20%. The reason behind so many firms having been located in the centre is – aside from the desire to be the dominant market player, which has already been discussed – the desire to earn more than the competitor. In real life, absolute income does not matter. What matters is whether our income is proportionate compared to the income of those around us, i.e. our family, friends or neighbours. People feel wealthy when they earn more than others do, and they feel poor when their financial status is worse than that of others. In the analyzed model, when one of the firms decided to abandon the equilibrium and move to the centre, the competition became fiercer due to the shorter distance between the competitors, leading to a decline in the profits for both players (by 5.2% for the leader, and by 15.8% for the follower). However, the profit of the firm which decided to make such a move was 11% higher than that of its competitor. The results show that in spite of being economically irrational, this move was made by nearly 20% of the players. Certainly a firm which is left in the boundary area considers moving to the centre, too. If they decide to do so, the profits of both players will decline even further. When two players for whom the market position is the top priority face each other, a ridiculous situation may occur, where instead of coming to an agreement, the firms will fight for location, which leads to a decline in their profits by 31.5% compared to the equilibrium point. As the experiment proved, this situation may actually happen. Among 22 pairs under study, as many as three decided to locate both firms in the centre of the analyzed area and did not change the location to the very end of the game.

The last analyzed situation involved two large competitors. Each of them is able to supply customers from roughly the entire area, which leads to an even fiercer market competition than before. This case is very interesting, as there is only one Nash equilibrium in this game, where both firms are centrally located. However, this location is not Pareto-effective. A Pareto-effective equilibrium is reached when the firms are partly dispersed and located two units away from each other, symmetrically



to the middle of the analyzed area. An example of such a location is shown in the figure below.

**Figure 10: A sample Pareto-effective equilibrium which is not a Nash equilibrium in a model representing two large firms**



There are four Pareto-effective equilibria in this model. They are the following locations:  $((3,2),(3,4))$ ,  $((3,4),(3,2))$ ,  $((2,3),(4,3))$  and  $((4,3),(2,3))$ . In each of these locations, the players will feel a unilateral temptation to move to the centre. This way their profit will increase by 7.9%, and the profit of their competitor will decline by 20%. Then the second player will also benefit from moving and locating their firm in the centre. This way the Nash equilibrium is reached, where profits are 16% lower than in the Pareto-effective equilibrium.

The key question in the case of two large firms is: will the firms manage to come to an agreement and reach a Pareto-effective equilibrium based on mutual trust without moving to the centre? Or will one of the players try to dominate the market, leading to a central location of both firms, i.e. an ineffective Nash equilibrium?

The previous results point to the latter. It is hard to believe that the players would be able to trust the rival and resist the urge to move to the centre under experimental conditions, where they cannot communicate with each other. The results of the experiment confirm this consideration. Almost 90% of the pairs eventually came to a Nash equilibrium, i.e. they decided to compete against each other by picking a central location. The remaining 10% did not reach any type of

equilibrium, making unilateral efforts to come to an agreement and to leave the ineffective central location. In the two final rounds of the experiment only slightly more than 5% of the players did not pick central locations. It means that the other half of the players who had not reached the Nash equilibrium decided to stay in the middle. This proves the thesis that it is impossible to make location decisions in a synchronized manner.

## **Summary**

The key conclusion from this analysis is that the importance of firm size for location decisions has been confirmed. As indicated by the equilibria obtained from the theoretical model and experimental results, small firms pick peripheral locations. By dispersing, they divide the market between themselves and do not need to compete. Thus they maximize their profits, which are as high as in a monopolistic market. With the increase in firm size, the competition becomes stronger and the dispersion is lower. The critical point is reached when large firms become concentrated in one spot in the middle of the area. Psychology is yet another factor which adds to the centripetal force; it is the desire to occupy the central location and to “dominate the market”. It even happens at the expense of a part of the profits. The results obtained in this paper can be very stimulating and lead at least to a change in the perception of economic phenomena. Firstly, the market size and the capability to saturate the market determine product differentiation. When none of the firms is able to serve the total market demand (within the meaning of both the location and product characteristics), the maximum differentiation principle will apply. For large firms, which are able to satisfy the total demand, competition leads to minimum product differentiation. Secondly, globalization processes may broaden the gaps between the suburbs and the metropolis – i.e. the divergence. When large firms emerge in national markets, the resulting competition may lead to firms locating their activities in the centre of the country. They will manufacture their products according to the minimum differentiation principle, which may aggravate the undesirable and adverse effects of globalization.

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