

Ryszard Kutner

My Activities

Ryszard Kutner: My Activities

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Welcome

HOME PAGE of Ryszard Kutner: Career

Born in Łódź, Poland, 15 April 1947, married with Anna Maria Sobolska-Kutner, one daughter Joanna Elżbieta Kutner-Pielaszek married with Roman Pielaszek, two grandsons Filip and Adam and one granddaughter Paulina

Tenure Professor (the title of professor of physical sciences from 2011 year)

Position: Professor on the Chair of Econo- and Sociophysics Group in Division of Biomedical Physics, Faculty of Physics, University of Warsaw, Pasteura 5, PL-02093 Warsaw, Poland

<http://www.fuw.edu.pl/home.html>

<http://brain.fuw.edu.pl/index.php/ekono-fizyka/>

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[tp://www.fuw.edu.pl/tl_files/informator/Ist/2015-2016/Ekonofizyka/Ist_2015_16.pdf](http://www.fuw.edu.pl/tl_files/informator/Ist/2015-2016/Ekonofizyka/Ist_2015_16.pdf)

Member of the Polish Physical Society (PPS) and European Physical Society (EPS)

Chairman of the Polish Physical Society Section of Physics in Economy and Social Sciences

<http://www.ptf.net.pl/pl/towarzystwo/sekcje/fens/zarzad/>

Coorganizer of the cyclic Polish Symposiums on Physics in Economy and Social Sciences systematically from 2004

<http://www.ptf.net.pl/pl/towarzystwo/sekcje/fens/>

Statistics (from scholar.google.com)

- total number of publications = 156
- h-index = 20
- i10-index = 31 (concerning papers' citations no less than 10 times)
- number of paper cited no less than 100 times = 6
- total number of citations = 1961
- number of citations for last 5 year = 469
- maksimal number of citations of a single paper = 265
- Attention: The corresponding results taken from the Web of Science reduces all above given results (to good approximation) by 10% except the total number of citations without self-citations, which reduce above given total number of citations by about 15%.

Academic experience

• Master thesis at University of Warsaw (1970)

• Research scientist at Faculty of Physics, University of Warsaw (1970-1977)

• PhD at Faculty of Physics University of Warsaw (1977)

• Post-Doc (on Henrich Hertz fund) in Kurt Binder and Klaus Kehr Group, Institut für Festkörperforschung, Kernforschungsanlage (now Forschungszentrum) Jülich, Germany, (1979-1981).

• Researcher in Kurt Binder and Klaus Kehr group, Institut für

Festkörperforschung, Kernforschungsanlage (now Forschungszentrum) Jülich, Germany, (1983-85).

• Habilitation at Faculty of Physics University of Warsaw (1985)

• Invited lecture in Wolfgang Dieterich group, Faculty of Physics, Konstanz University, Germany (1991).

• Researcher in Kazuo Kitahara group, Department of Applied Physics, Tokyo Institute of Technology, Japan (1991)

• Visiting professor: Konstanz University (1993-1995).

• Invited lecture in Joel Lebowitz group, Theoretical Condensed Matter Physics, Center for Mathematical Sciences Research, Rutgers University, US (1999)

• Invited lecture in Eugene Stanley group, Center for Polymer Studies, Department of Physics, Boston University (1999).

• Invited lecture in Wolfgang Dieterich group, Faculty of Physics, University of Konstanz, Germany (2002)

• Invited lecture in Katarzyna Sznajd-Weron group, Institute of Physics, Wrocław University (2010).

• Further several invited talks and lectures on domestic and international conferences, symposiums and schools.

• Referee and distinguished referee of several scientific international journals (Physica A, The European Physical Journal B, Physical Review E, Physical Review Letters, European Journal of Physics, Journal of Physics A, Physics Letters A, New Journal of Physics, European Physics Letters, ...)

• Several systematic lectures for graduate and PhD students from Faculty of Physics since about two decades - see USOS web:

ht-

[tps://usosweb.uw.edu.pl/kontroler.php?_action=katalog2/osoby/pokaZOsobe&os_id=3025](https://usosweb.uw.edu.pl/kontroler.php?_action=katalog2/osoby/pokaZOsobe&os_id=3025)

• Promoted dozens bachelors, masters theses, doctoral dissertations - see USOS web (login and password delivered separately):

<https://apd.uw.edu.pl>

• Currently, a supervisor of 4 PhD-students and several master and bachelor ones.

<http://old.ptf.net.pl/fens/>

• Reviewer dozens of doctoral, graduate and undergraduate theses and reviewer of Centralna Komisja do Spraw Stopni i Tytułów in a few habilitation procedures as well chairman in such procedures as a referee in procedures concerning the title of professor.

• The member of Kapituła d/s Nagród Naukowych Polskiego Towarzystwa Fizycznego.

Lectures and seminars (from 2005 year to now) https://usosweb.uw.edu.pl/kontroler.php?_action=katalog2/osoby/pokaZOsobe&os_id=3025

Winter semester

1. Symulacje komputerowe w fizyce z przykładami (kod w USOS: 1101-5Eko11), 30h, wykład specjalistyczny.
2. Niegaußowskie procesy stochastyczne w naukach przyrodniczych z elementami ekono- i socjofizyki (kod w USOS: 1101-5Eko12), 30h, wykład specjalistyczny.

3. Seminarium z ekono- i socjofizyki II (kod w USOS: 1101-5sESF1) 30h, seminarium specjalistyczne.

Spring semester

1. Metody fizyki w ekonomii – wprowadzenie (kod w USOS: 1101-4Eko23) 30h, wykład specjalistyczny.
2. Fizyka dnia codziennego (kod w USOS: 1101-FDC-OG), 30h, wykład ogólnouniwersytecki.
3. Wprowadzenie do fizyki złożoności. Fizyka statystyczna sieci złożonych (kod w USOS: 1100-WFZ-OG), 30h, wykład ogólnouniwersytecki.
4. Seminarium z ekono- i socjofizyki I(kod w USOS:1101-5sESF1),30h, seminarium specjalistyczne.
5. Proseminarium z fizyki układów złożonych B2+ (kod w USOS: 1101-5Eko25), 30h, proseminarium specjalistyczne.

Teaching materials for students are placed under the address: <http://www.fuw.edu.pl/materialy-IV-V-rok.html>

Organizational activity

- Organizer of Postgraduate Studies of Physics and Astronomy at Faculty of Physics University of Warsaw (in academic year 1988-89) and its head from year 1988 to 1993.
- Organizer (in 2006) and chairman of Physics in Economy (Econophysics) speciality on Faculty of Physics University of Warsaw (from 2006 to now).
- Coorganizer, member of scientific committees of domestic and international conferences, and chair of scientific committee of many scientific conferences, for instance, <http://science24.com/event/fens2015/> Currently, the main organizer of the international Econophysics Colloquium 2017 in Warsaw.

Awards

- Individual award of the third degree of the MNSWiT in 1978 year for my doctoral thesis.
- Individual award of the second degree for achievements in the scientific research of the Rector of the University of Warsaw, 1979 year.
- Award of the Director of the IFD WF UW for organizing and directing the Postgraduate Studies for Teachers of Physics, 1982 year.
- Dean WF UW Award for organizational activities relating to computerization of the WF UW, 1989 year.
- Editor's Award journal Physics in the School for a series of articles: *Kącik fizyki komputerowej*, 1990 year.
- Thanks from the Board of the National Children's Fund for long-term cooperation and in particular for the teaching of youth remarkably gifted, 1993 year.
- Jubilee Award of the Rector of the University of Warsaw on the occasion of the 20th anniversary of my professional activity, 1995 year.
- Praise the Rector of the University of Konstanz (Germany) for active participation in the Open Days University, 1995 year.
- Thanks Editor of the international Journal of Physics for working as a reviewer in 1995-99.
- Praise Director of the Institute of Theoretical Physics, University

of Wrocław prof. dr. George Łukierskiego for the joint organization of the Symposium XI Max Born and exemplary fulfillment of the obligations of one of the two directors Symposium (the other was Professor. A. Pękalski of IFT UWr), 1998 year.

- Organisers II Memorial Festival of Science in 1998. For the teaching of young people.
- Edited by me (as a leading editor and co-author) PWN Multimedia Encyclopedia of booklet 14 with CD titled: Technique, occupied in 2001 first place in the ranking, the biggest on the Polish market computer magazine: Computer and World, Publishing Axel Springer Poland, among eleven located on the Polish market of multimedia encyclopedia.
- Jubilee Award of the Rector of the University of Warsaw on the occasion of the 30th anniversary of professional activity, 2004 year.
- International Awards of the Journal of Physics for working as a reviewer in the years 1995-2001 and 2001-2005.
- Awards Manager Initiative entitled: "New Emerging Science and Technology", which is part of the 6th EU Framework Programme of the European Union, for reviewing nine international projects in 2004 and 2006.
- Memorial organizers of the XIII Science Festival in 2008 year for a lecture presentation.
- Memorial organizers of the XIV Festival of Science in 2010 year for a lecture presentation.
- Letter of commendation of the President of the Polish National Bank issued in 2011 for promising scientific activity of my research team, which completed a research grant obtained by my team from the Economic Institute of the NBP for 2010-2011.
- Distinguished referee of Physica A and The European Physical Journal B in 2014.

Interest

General interest

- Various random walks (Gaussian and non-Gaussian, in discrete and continuous times) and diffusions (normal and anomalous) on regular and on statically and dynamically disordered lattices and networks.
- Complex systems (in physics, econophysics, sociophysics, geophysics, biomedical physics, economy including finance, social science).
- Evolving complex networks and their applications in different branches of science.
- Equilibrium and non-equilibrium statistical mechanics and thermodynamics (theory of phase transitions, multifractality) and their applications.
- Stochastic dynamics, Gaussian and non-Gaussian stochastic processes and their applications
- Applications of Extreme Value Theory
- Applications of Catastrophe Theory
- Power-laws, universality (in physics and beyond)
- Non-Debye relaxation
- Numerical simulations in physics and beyond

Current interest

Inter- and cross-disciplinary science

Physics in finance, economy and social sciences

- Physics of complex systems and pattern formation in spatio-temporal structures, dynamical system theory, pattern recognition, self-organized criticality, prediction of complex systems, time series analysis.
- Prediction of crises and extreme events in complex systems (with applications to finance, economics, sociology, biology, medicine, earthquakes, rupture, climate change).
- Finance and economics: predictability in financial markets, theory of bubbles and crashes and tests, large risks and tail dependence, theory of derivatives, portfolio optimization, trading strategies, insurance, macro-economics, agent-based models, market microstructure

Physics in biomedicine

- Prediction of epileptic seizure and asthma attack

Geophysics

- Study of earthquakes, ruptures, climate changes.

Scientific grants

In the last decade a head of few domestic and international scientific grants, e.g.,

1. Grant Promotorski MNiSW No. GR-2392 (2006-07).
2. Polish Research Grant No. 119 obtained within the First Competition of the Committee of Scientific Research organized by the National Bank of Poland (2010-11).

3. *Study and Modelling of Households Incomes in Different Countries and Regions in Europe*, Research Grant of EC EUROSTAT EU Statistics on Income and Living Conditions (SILC) No. EU-SILC/2010/26, since 2010.

Moreover, in the years 1988-2005 I was the head of the four research grants obtained in winning competitions of the MNiSW (or KBN). All grants received high marks final including one excellent.

References

Science24 (list of publications and their abstracts): <http://science24.com/event/econo-erka/journal/?item=3>

Google Scholar (list of publications ordered according citations, citation profile): <https://scholar.google.pl/citations?user=rIeefdQAAAAJ&hl=en&oi=ao>

Research Gate: *List of publications with abstracts, Skills, Research Expertise, Topics, and Disciplines (Physics and Social Science), Research Connections:* https://www.researchgate.net/profile/Ryszard_Kutner/

Most significant publications: 2005-2016

[1] M. Kozłowska, M. Denys, M. Wiliński, G. Link, T. Gubiec, T. R. Werner, **R. Kutner**, and Z.R. Struzik (Tokyo Univ.): *Dynamic bifurcations on financial markets*, Chaos, Solitons & Fractals, <http://dx.doi.org/10.1016/j.chaos.2016.03.005>(2016), w druku.

[2] M. Denys, M. Jagielski, T. Gubiec, **R. Kutner**, H.E. Stanley (Boston Univ.): *Universality of market superstatistics*, w recenzji, arXiv:1509.06315 [q-fin.ST].

[3] M. Wiliński, B. Szewczak, T. Gubiec, **R. Kutner**, Z.R. Struzik (Tokyo Univ.): *Temporal condensation and dynamic lambda-transition within the complex network: an application to real-life market evolution*, Eur. Phys. J. B 88 (2015), 1.

[4] M. Jagielski, R. Duczmal, **R. Kutner**, *Income Distribution in the European Union Versus in the United States*, Physica A 433 (2015), 36.

[5] M. Wiliński, A. Sienkiewicz, T. Gubiec, **R. Kutner**, Z.R. Struzik (Tokyo Univ.): *Structural and topological phase transitions on the German Stock Exchange*, Physica A 392 (2013), 5963.

[6] M. Jagielski, **R. Kutner**: *Modelling of income distribution in the European Union with the Fokker–Planck equation*, Physica A 392 (2013), 2130.

[7] T. Werner, T. Gubiec, **R. Kutner**, D. Sornette (ETH Zürich): *Modeling of super-extreme events: An application to the hierarchical Weierstrass-Mandelbrot Continuous-Time Random Walk in Power-laws in real systems and beyond*, The European Journal of Physics Special Topics 205 (2012), 27.

[8] T. Gubiec, **R. Kutner**: *Backward jump continuous-time random walk: An application to market trading*, Physical Review E 82 (2010), 046119.

[9] A. Kasprzak, **R. Kutner**, J. Perelló (Univ. of Barcelona), J. Masoliver (Univ. of Barcelona): *Higher-order phase transitions on financial markets*, The European Journal of Physics B 76 (2010), 513.

[10] J. Perelló (Univ. of Barcelona), J. Masoliver (Univ. of Barcelona), A. Kasprzak, and **R. Kutner**: *Model for interevent times with long tails and multifractality in human communications: An application to financial trading*, Physical Review E 78 (2008), 036108.

[11] M. Kozłowska, A. Kasprzak, **R. Kutner**: *Fractional Market Model and its Verification on the Warsaw Stock Exchange*, Interna-

tional Journal of Modern Physics C 19 (2008), 453.

[12] M. Kozłowska, **R. Kutner**: *Anomalous transport and diffusion versus extreme value theory*, Physica A 357 (2005), 282.

[13] J. Hurkała, M. Gall, **R. Kutner**, M. Maciejczyk: *Real-time numerical simulation of the Carnot cycle*, European Journal of Physics 26 (2005), 673.

International scientific cooperation

1) Prof. Zbigniew R. Struzik, The Tokyo University

2) Prof. Didier Sornette, ETH Zurich.

3) Research group of prof. Jaume Masoliver, University of Barcelona.

4) Prof. H. Eugene Stanley, Boston University.

Editor of Conference Proceedings and Special Issues

R. Kutner, A. Pękalski, K. Sznajd-Weron (eds.): *Anomalous Diffusion. From Basics to Applications*. Lecture Notes in Physics Vol. 519, Springer-Verlag, Berlin 1999.

R. Kutner, J. Hołyst (eds): *First Polish Symposium on Econo- and Sociophysics*, Acta Physica Polonica B 36(8), 2397 (2005).

D. Grech, J. Hołyst, W. Kamiński, **R. Kutner**, A. Orłowski (eds): *Proceedings of the 7th Polish Symposium of Physics in Economy and Social Sciences*, Acta Physica Polonica A 127(3-A), A-1 (2015).

S. Drożdż, D. Grech, **R. Kutner**, R. Rak (eds.): *Proceedings of the 8th Polish Symposium of Physics in Economy and Social Sciences*, Acta Physica Polonica A (2016), in print.

R. Kutner, J. Masoliver (eds): *Continuous Time Random Walk still trendy: Fifty-years history, current state, and outlook*, The European Physical Journal B (2016) Special Issue, in print.

Publications

Abstracts

in author alphabetical order

Radio Broadcast

Rozmowa o Ekonofizyce - Akademickie Radio Kampus

Ryszard Kutner

Warsaw University, Faculty of Physics, Hoża 69, Warszawa 00-681, Poland

e-mail: erka@fuw.edu.pl

Rozmowa dotyczy intrygującego związku szerokokorozumianej fizyki z ekonomią. Dostarcza roboczej definicji ekonofizyki wraz z jej metodologią i technikami. Zawiera szereg praktycznych informacji o specjalności ekonofizycznej na Wydziale Fizyki Uniwersytetu Warszawskiego. Ponadto, pozwala dostrzec kierunki rozwoju ekonofizyki na świecie.



Article

Higher-order phase transitions on financial markets

Andrzej Kasprzak¹, Ryszard Kutner¹, Josep Perello², Jaume Masoliver²

1. *Warsaw University, Faculty of Physics, Hoża 69, Warszawa 00-681, Poland* **2.** *Universitat de Barcelona, Departament de Física Fonamental, Diagonal 647, Barcelona 08028, Spain*

e-mail: erka@fuw.edu.pl

Statistical and thermodynamic properties of the anomalous multifractal structure of random interevent (or intertransaction) times were thoroughly studied by using the extended continuous-time random walk (CTRW) formalism of Montroll, Weiss, Scher, and Lax. Although this formalism is quite general (and can be applied to any interhuman communication with nontrivial priority), we consider it in the context of a financial market where heterogeneous agent activities can occur within a wide spectrum of time scales. As the main general consequence, we found (by additionally using the Saddle-Point Approximation) the scaling or power-dependent form of the partition function, $Z(q)$. It diverges for any negative scaling

powers q (which justifies the name anomalous) while for positive ones it shows the scaling with the general exponent $\tau(q)$. This exponent is the nonanalytic (singular) or noninteger power of q , which is one of the pillar of higher-order phase transitions. In definition of the partition function we used the pausing-time distribution (PTD) as the central one, which takes the form of convolution (or superstatistics used, e.g. for describing turbulence as well as the financial market). Its integral kernel is given by the stretched exponential distribution (often used in disordered systems). This kernel extends both the exponential distribution assumed in the original version of the CTRW formalism (for description of the transient photocurrent measured in amorphous glassy material) as well as the Gaussian one sometimes used in this context (e.g. for diffusion of hydrogen in amorphous metals or for aging effects in glasses). Our most important finding is the third- and higher-order phase transitions, which can be roughly interpreted as transitions between the phase where high frequency trading is most visible and the phase defined by low frequency trading. The specific order of the phase transition directly depends upon the shape exponent α defining the stretched exponential integral kernel. On this basis a simple practical hint for investors was formulated.

The European Journal of Physics B 76, 513–527 (2010).

Article

Random walk on a random walk

Ryszard Kutner¹, Klaus W. Kehr²

1. *Faculty of Physics, University of Warsaw (FPUW), Pasteura 5, Warsaw 02-093, Poland* **2.** *Institut für Festkörperforschung, Forschungszentrum Jülich (FZJ), Wilhelm-Johnen-Straße, Jülich 52428, Germany*

e-mail: erka@fuw.edu.pl

The authors investigate the random walk of a particle on a one-dimensional chain which has been constructed by a random-walk procedure. Exact expressions are given for the mean-square displacement and the fourth moment after n steps. The probability density after n steps is derived in the saddle-point approximation, for large n . These quantities have also been studied by numerical simulation. The extension to continuous time has been made where the particle jumps according to a Poisson process. The exact solution for the self-correlation function has been obtained in the Fourier and Laplace domain. The resulting frequency-dependent diffusion coefficient and incoherent dynamical structure factor have been discussed. The model of random walk on a random walk is applied to self-diffusion in the concentrated one-dimensional lattice gas where the correct asymptotic behavior is found.

Physica A 110, 535 (1982).

Temporal condensation and dynamic λ -transition within the complex network: an application to real-life market evolution

Mateusz J. Wiliński¹, Bartłomiej Szewczak², Tomasz Gubiec¹, Ryszard Kutner¹, Zbigniew R. Struzik³

1. Uniwersytet Warszawski, Wydział Fizyki, ul. Pasteura 5, Warszawa 02-093, Poland **2.** University of Warsaw, Faculty of Physics, Institute of Experimental Physics (IFDUW), Hoża 69, Warsaw 00-681, Poland **3.** The University of Tokyo, Bunkyo-ku, Tokyo 113-8655, Japan

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We fill a void in merging empirical and phenomenological characterisation of the dynamical phase transitions in complex networks by identifying and thoroughly characterising a triple sequence of such transitions on a real-life financial market. We extract and interpret the empirical, numerical, and analytical evidences for the existence of these dynamical phase transitions, by considering the medium size Frankfurt stock exchange (FSE), as a typical example of a financial market. By using the canonical object for the graph theory, i.e. the minimal spanning tree (MST) network, we observe: (i) the (initial) dynamical phase transition from equilibrium to non-equilibrium nucleation phase of the MST network, occurring at some critical time. Coalescence of edges on the FSE's transient leader (defined by its largest degree) is observed within the nucleation phase; (ii) subsequent acceleration of the process of nucleation and the emergence of the condensation phase (the second dynamical phase transition), forming a logarithmically diverging temporal λ -peak of the leader's degree at the second critical time; (iii) the third dynamical fragmentation phase transition (after passing the second critical time), where the λ -peak logarithmically relaxes over three quarters of the year, resulting in a few loosely connected sub-graphs. This λ -peak (comparable to that of the specific heat vs. temperature forming during the equilibrium continuous phase transition from the normal fluid I ^4He to the superfluid II ^4He) is considered as a prominent result of a non-equilibrium superstar-like superhub or a dragon-king's abrupt evolution over about two and a half year of market evolution. We capture and meticulously characterise a remarkable phenomenon in which a peripheral company becomes progressively promoted to become the dragon-king strongly dominating the complex network over an exceptionally long period of time containing the crash. Detailed analysis of the complete trio of the dynamical phase transitions constituting the λ -peak allows us to derive a generic nonlinear constitutive equation of the dragon-king dynamics describing the complexity of the MST network by the corresponding inherent nonlinearity of the underlying dynamical processes.

The European Journal of Physics B (2015) 88:34, 1-15.

Backward jump continuous-time random walk: An application to market trading

Tomasz Gubiec, Ryszard Kutner

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e-mail: erka@fuw.edu.pl

The backward jump modification of the continuous-time random walk model or the version of the model driven by the negative feedback was herein derived for spatiotemporal continuum in the context of a share price evolution on a stock exchange. In the frame of the model, we described stochastic evolution of a typical share price on a stock exchange with a moderate liquidity within a high-frequency time scale. The model was validated by satisfactory agreement of the theoretical velocity autocorrelation function with its empirical counterpart obtained for the continuous quotation. This agreement is mainly a result of a sharp backward correlation found and considered in this article. This correlation is a reminiscence of such a bid-ask bounce phenomenon where backward price jump has the same or almost the same length as preceding jump. We suggested that this correlation dominated the dynamics of the stock market with moderate liquidity. Although assumptions of the model were inspired by the market high-frequency empirical data, its potential applications extend beyond the financial market, for instance, to the field covered by the Le Chatelier-Braun principle of contrariness.

PHYSICAL REVIEW E 82, 046119 (2010).

Comparative Analysis of Income Distributions in the European Union and the United States

Maciej Jagielski, Rafał J. Duczmal, Ryszard Kutner

University of Warsaw, Faculty of Physics, Institute of Experimental Physics (IFDUW), Hoża 69, Warsaw 00-681, Poland

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We prove that the most refined approach - our extension of the Yakovenko et al. model - is a universal in the sense that it well describes both household incomes in the European Union and the individual incomes in the United States for all income social classes. This prove was based on our comparative study of various kinds of incomes. The study constitutes a basis for the finding of an impact of the recent world-wide financial crisis on the volatility of various temporary Pareto exponents and on other parameters of the model.

Acta Physica Polonica A 127, A-75 (2015).

Diffusion in one-dimensional bosonic lattice gas

Ryszard Kutner¹, Klaus W. Kehr², Wolfgang Renz², Radosław Przeniosło¹

1. Faculty of Physics, University of Warsaw (FPUW), Pasteura 5, Warsaw 02-093, Poland **2.** Institut für Festkörperforschung, Forschungszentrum Jülich (FZJ), Wilhelm-Johnen-Straße, Jülich 52428, Germany

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A one-dimensional lattice-gas model with order preservation is considered where the occupation probabilities of sites correspond to Bose statistics as a consequence of the prescribed dynamics. The master equation for the particle-cluster dynamics at the sites is formulated. The corresponding continuum nonlinear diffusion equation is derived for the space- and time-dependent concentration fluctuations. The equation can be regarded, in the presence of a drift force, as the Burgers equation when terms irrelevant in the sense of renormalization-group ideas are neglected. Collective centre-of-mass and tagged-particle diffusion are investigated by numerical simulations and the results agree with the analytical derivations. Subdiffusive behaviour of the mean-square displacement of tagged particles and normal collective and centre-of-mass diffusion are observed when no bias is present. The dispersion of the centre-of-mass displacement exhibits superdiffusive behaviour in the case of mean drift of the particles. Discrepancies of about 20% between the numerically determined superdiffusion coefficients and the predictions of the mode-coupling theory are found and discussed.

Journal of Physics A: Mathematical and General 28 (1995), 923.

Diffusion in concentrated lattice gases. III. Tracer diffusion on a one-dimensional lattice

Henk van Beijeren¹, Klaus W. Kehr², Ryszard Kutner³

1. Institute of Theoretical Physics, Utrecht University (ITPUU), Leuvenlaan 4, Utrecht 3584 CE, Netherlands **2.** Institut für Festkörperforschung, Forschungszentrum Jülich (FZJ), Wilhelm-Johnen-Straße, Jülich 52428, Germany **3.** Faculty of Physics, University of Warsaw (FPUW), Pasteura 5, Warsaw 02-093, Poland

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The dynamical process of the diffusion of tagged particles in a one-dimensional concentrated lattice gas is investigated. The particles are noninteracting except that double occupancy is forbidden. The mean-square displacement of a tagged particle is calculated for all times by an approximate theory and compared to results from Monte Carlo simulations. The overall agreement is quite good. For an infinite chain and for large time t the mean-square displacement is found to increase proportionally to $t^{1/2}$ in agreement with existing results. For periodic chains it increases as $2D_{\text{tr}} t$ for large times, with a coefficient of tracer diffusion D_{tr} inversely proportional to the number of particles on the chain. This, too, is in agreement with the results of older calculations. In the case of hard reflecting walls finally the mean-square displacement asymptotically ap-

proaches a constant, which can be calculated simply.

Physical Review B 28, 5711 (1983).

Diffusion in concentrated lattice gases. Self-diffusion of noninteracting particles in three-dimensional lattices

Klaus W. Kehr², Ryszard Kutner¹, Kurt Binder³

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The dynamical process of the diffusion of tagged particles in concentrated lattice gases is investigated. The lattice is fcc and the particles are noninteracting except that double occupancy is forbidden. The self-diffusion coefficient and the waiting-time distributions for two consecutive jumps of a tagged particle are calculated numerically by Monte Carlo methods. The time-dependent correlations between consecutive jumps are analyzed in detail, especially the initially increased rate for backward jumps of the tagged particle. The self-correlation function of a diffusing particle is also calculated by a correlated-jump model, which represents a generalization of the usual treatment of self-diffusion to frequency dependence and to finite concentrations. The resulting diffusion coefficient agrees well with the direct numerical calculations. Some other analytical predictions are also tested. The incoherent dynamical scattering function shows the effect of correlations at small frequencies and mean-field behavior at high frequencies.

Physical Review B 23, 4931 (1981).

Hierarchical spatio-temporal coupling in fractional wanderings. (I) Continuous-time Weierstrass flights

Ryszard Kutner

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The one-dimensional continuous-time Weierstrass flights (CTWF) model is considered in the framework of the nonseparable continuous-time random walks formalism (CTRW). A novel spatio-temporal coupling is introduced by assuming that in each scale the probability density for the flight and for waiting are joined. Hence, we treat the spatio-temporal relations in terms of the self-similar structure of the Weierstrass process. This (stochastic) structure is characterized by the spatial fractional dimension $1/\beta$ representing the flights and the temporal one $1/\alpha$ representing the waiting. Time was assumed here as the only independent truncation range. In this work we study the asymptotic properties of the CTWF model. For example, by applying the method of steepest descents we obtained the particle propagator in the approximate scaling form,

$$P(X, t) \sim t^{-\eta(\alpha, \nu, \beta)} / 2 \mathcal{F}(\xi)$$

where the scaling function

$$\mathcal{F}(\xi) = \xi^{\bar{\nu}(\alpha, \nu, \beta)} \exp(-\text{const.}(\alpha, \beta) \xi^{\nu(\alpha, \nu, \beta)})$$

while the scaling variable $\xi = |X|/t^{\eta(\alpha, \beta)/2}$ is large. The principal result of our analysis is that the exponents η , ν , and $\bar{\nu}$ depend on more fundamental ones, α and β , what leads to a novel scaling. As a result of competition between exponents α and β an enhanced, dispersive or normal diffusion was recognized in distinction from the prediction of the separable CTRW model where the enhanced diffusion is lost and the dispersive one is strongly limited. It should be noted that we compare here partially thermalized versions of both approaches where some initial fluctuations were also included in agreement with the spirit of the theory of the renewal processes. Having the propagator we calculated, for example the particle mean-square displacement and found its novel asymptotic scaling with time for enhanced diffusion, given by $\propto t^{1+\frac{2\nu}{\alpha}(\beta-1)}$, in distinction from its diverging for $\beta < 2$ within the separable CTRW model. Our version of the nonseparable CTRW approach, i.e. the CTWF model or renewed continuous-time Lévy flights (CTLF), offers a possibility to properly model the time-dependence for any fractional (critical) wandering of jump type. A similar, though mathematically more complicated analysis is also applied in Part II to the Weierstrass walks (WW) which leads to a novel version of the Lévy walks (LW) process.

Physica A 264, 84 (1999).

Book

Anomalous Diffusion: From Basics to Applications

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This collection of articles gives a nice overview of the fast growing field of diffusion and transport. The area of non-Browman statistical mechanics has many extensions into other fields like biology, ecology, geophysics etc. These tutorial lectures address e.g. Lévy flights and walks, diffusion on metal surfaces or in superconductors, classical diffusion, biased and anomalous diffusion, chemical reaction diffusion, aging in glassy systems, diffusion in soft matter and in nonsymmetric potentials, and also new problems like diffusive processes in econophysics and in biology.

Lecture Notes in Physics Vol. 519 (1999).

Article

Diffusion in concentrated lattice gases IV. Diffusion coefficient of tracer particle with different jump rate

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We have investigated tracer diffusion in a three-dimensional lattice gas of arbitrary concentration by Monte-Carlo simulation. The tracer particle has a jump rate which differs from that of the other particles. Results are presented for the tracer correlation factor for various concentrations and ratios of the jump rates. At high particle concentration, Manning's theory for impurity diffusion is applicable. The critical behaviour near the percolation threshold of the vacancies is compared with the scaling laws given by Gefen, Aharony and Alexander and their generalization to a finite jump rate of the other particles. A recent theory proposed by Nakazato and Kitahara provides a good description of the data at small particle concentration.

Philosophical Magazine A 48, 199 (1983).

Article

Correlated hopping in honeycomb lattice: tracer diffusion coefficient at arbitrary lattice gas concentration

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The tracer diffusion coefficient is investigated for a correlated diffusion of a tracer, which is a particle of the lattice gas. The lattice gas consists of particles of arbitrary concentration hopping on a two-dimensional honeycomb lattice, which interact only as repulsive point hard cores. Exchange between different particles is excluded, and any particle can jump only to the nearest-neighbour empty sites. The tracer diffusion coefficient is studied in a wide range of concentrations by the Monte Carlo simulations and compared with different theoretical predictions. It is shown that the Nakazato and Kitahara theory (1980) as well as the Tahir-Kheli and Elliot (1982) one give a good description of the simulation data. Moreover, it is proved that the traditional tracer diffusion coefficient that results from direct correlations over two consecutive jumps of a tracer is in distinct disagreement with the simulation data. Therefore a generalised expression for the tracer diffusion coefficient is derived; it includes direct correlations over an arbitrary number of jumps expressed in terms of integrals over 'waiting-time distributions'. The description of the data is much improved already by taking into account direct correlations over four jumps. Hence, it is concluded that the Nakazato and Kitahara theory as well as that of Tahir-Kheli and Elliott must include direct correlations over several consecutive jumps of a tracer-particle, which is not evident from the formalisms of these authors. Journal of Physics C: Solid State Physics 18 (1985), 6323.

Diffusion in concentrated lattice gases. V. Particles with repulsive nearest-neighbor interaction on the face-centered-cubic lattice

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The diffusion of particles in concentrated lattice gases is studied by Monte Carlo methods, assuming a fcc lattice with repulsive nearest-neighbor interaction. Particular attention is paid to the influence of ordering on the diffusion properties, since the model has ordered superstructures at low temperatures T near the stoichiometric concentrations $c=14, 12$, and 34 . Both the self-diffusion of tagged particles and the collective diffusion by which concentration fluctuations decay are obtained. In the ordered regions both diffusivities are rather small due to a strong decrease of the effective jump rate of the particles. The correlation factor $f(T,c)$ for self-diffusion has a pronounced non-monotonic concentration dependence for low temperatures. This is interpreted by reducing the problem near $T=0$ and the limits $c \rightarrow 14-, 12-$ to an effective single-vacancy problem, and $c \rightarrow 14+, 12+$ to an effective single-particle problem. Other lattices are briefly discussed.

Physical Review B 28, 1846 (1983).

Thermal neutron scattering from a hydrogen-metal system in terms of a general multi-sublattice jump diffusion model—I: Theory

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A Multi-Sublattice Jump Diffusion Model (MSJD) for hydrogen diffusion through interstitial-site lattices is presented. The MSJD approach may, in principle, be considered as an extension of the Rowe *et al.*[1] model. Jump diffusion to any neighbours with different jump times which may be asymmetric in space is discussed. On the basis of the model a new method of calculating the diffusion tensor is advanced. The quasielastic, double differential cross section for thermal neutron scattering is obtained in terms of the MSJD model. The model can be used for systems in which interstitial jump diffusion of impurity particles occurs. In Part II the theoretical results are compared with those for quasielastic neutron scattering from the αNbH_x system.

Journal of Physics and Chemistry of Solids 38, 741 (1977).

Susceptibility and transport coefficient in a transient state on a one-dimensional lattice. I. Extended linear response and diffusion

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In this paper (Part I) we extend the linear response analysis to calculate the complex dynamic susceptibility and the complex dynamic mobility/conductivity for a system in a *transient* state relaxing to equilibrium. This analysis has a meaning in the intermediate time and frequency region; for example, for solvation dynamics and non-linear relaxation in electrolyte systems. The discussed situation differs from the one considered by the usual linear response theory since our system during its relaxation is still under the action of an external force. It relaxes and simultaneously somehow *responds* to the applied force. As a test model we assume a single-particle one-dimensional random walk on a lattice in an *inhomogeneous*

periodic potential where a transient state is created by a nonequilibrium initial probability distribution. Using spectral analysis we derived spectral and summed formulas for the above mentioned dynamic quantities. The feature which distinguishes the present result is a force-dependence, since summed formulas depend, in general, on the external force but do not depend on its amplitude. The spectral analog of the dissipation-fluctuation theorem of the first kind was derived. In addition, a time- and frequency-dependent diffusion coefficient was studied. As a striking effect, we found a nonmonotonic frequency and time dependence of transport coefficients. The reason for this effect is a competition between the terms belonging to the different modes and contributing to the spectral analog of the current-current correlation functions with opposite signs, in contrast to the situation in equilibrium. The external force can additionally increase this effect. The dc values are well described by the usual linear response expressions for the thermalized system. The relation between the dynamic mobility and the frequency-dependent diffusion coefficient is still an open question for the system in a transient state. The systematic numerical studies of the results were performed in Part II by the Exact Enumeration method and by Monte Carlo simulation.

Physica A 224 (1996), 558.

Modeling of income distribution in the European Union with the Fokker-Planck equation

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Herein, we applied statistical physics to study incomes of three (low-, medium- and high-income) society classes instead of the two (low- and medium-income) classes studied so far. In the frame of the

threshold nonlinear Langevin dynamics and its threshold Fokker–Planck counterpart, we derived a unified formula for description of income of all society classes, by way of example, of those of the European Union in years 2006 and 2008. Hence, the formula is more general than the well known formula of Yakovenko et al.. That is, our formula well describes not only two regions but simultaneously the third region in the plot of the complementary cumulative distribution function vs. an annual household income. Furthermore, the known stylised facts concerning this income are well described by our formula. Namely, the formula provides the Boltzmann–Gibbs income distribution function for the low-income society class and the weak Pareto law for the medium-income society class, as expected. Importantly, it predicts (to satisfactory approximation) the Zipf law for the high-income society class. Moreover, the region of medium-income society class is now distinctly reduced because the bottom of high-income society class is distinctly lowered. This reduction made, in fact, the medium-income society class an intermediate-income society class.

Physica A 392 (2013), 2130.

Article

Diffusion in concentrated lattice gases. VI. Tracer diffusion on two coupled linear chains

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Tagged-particle diffusion is investigated on a system of two coupled linear chains with an arbitrary ratio of the jump rates for jumps to unoccupied neighboring sites in the two principal directions. The mean-square displacement of a tagged particle is investigated both theoretically and by Monte Carlo simulations; the agreement between the results of these two approaches is found to be very satisfactory. Also the theoretical and simulated correlation factors, determining the coefficient of tracer diffusion, are always found to agree within a few percent. At small jump rate ratios, the mean-square displacement exhibits an intermediate t^β power-law behavior, with $12 < \beta < 1$, resulting from an incomplete changeover to a purely one-dimensional $t^{1/2}$ power law.

Physical Review B 30, 4382 (1984)

Report on Foundation and Organization of Econophysics Graduate Courses at Faculty of Physics of University of Warsaw and Department of Physics and Astronomy of the Wrocław University

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Two different, working examples of organization of econophysics graduate courses at the Faculty of Physics, University of Warsaw and the Department of Physics and Astronomy of the Wrocław University are considered. In the first example we have a system where the interdisciplinary, econo-physical education begins only after three years study of physics. Within this system the M.Sc. as well as Ph.D. theses in econophysics are conducted only at the Faculty of Physics. In the second example the B.Sc. theses in econophysics are accomplished in the Department of Physics and Astronomy again after three years study but higher degrees can be prepared either in physics in the Institute of Theoretical Physics or in economy in the Institute of Economical Sciences. M.Sc. and Ph.D. theses can also be conducted. For both examples, the graduate students of econophysics are obliged to participate in traditional (typical) economical lectures and trainings which are offered them by economical departments while lectures and trainings (tutorials and/or laboratory classes) in econophysics are offered them by physics departments themselves. Thus Poland is one of a few countries, where so modern interdisciplinary knowledge is systematically offered to students.

Acta Physica Polonica-Series A General Physics 114 (2008), 637.

Article

Higher-order analysis within Weierstrass hierarchical walks

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We developed the Weierstrass walks (WW) model which describes both stationary and non-stationary stochastic time series. This model is a kind of Lévy walks, where we assume a hierarchical, self-similar in a stochastic sense, spatio-temporal representation of main probability densities. We consider a fractional random walk of a walker having, in general, different velocities between successive turning points. The WW model makes it possible to analyze the structure of the Hurst exponent. The analysis uses both the diffusion and the super Burnett coefficients. We constructed the diffusion phase diagram which distinguishes regions occupied by classes of different universality. We study only such classes which are characteristic of stationary situations. We thus have a model ready for describing of time series presented in the form of moving averages.

Income distribution in the European Union versus in the United States

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We prove that the refined approach -- our extension of the Yakovenko et al. formalism -- is universal in the sense that it describes well both household incomes in the European Union and individual incomes in the United States for all income social classes. This formalism, supplemented in this work by the entropy analysis, allowed the study of the impact of the recent world-wide financial crisis on the annual incomes of different income social classes. Hence, we find the most painful impact of the crisis on incomes of all income social classes. Furthermore, we indicate the existence of a possible market crisis precursor.

Physica A: Statistical Mechanics and its Applications, Volume 433, p. 36-41.

Stochastic simulations of time series within Weierstrass–Mandelbrot walks

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In the present work we extend Levy walks to allow the velocity of the walker to vary. We call these extended Levy walks Weierstrass–Mandelbrot walks. This is a generalized model of the Levy walk type which is still able to describe both stationary and non-stationary stochastic time series by treating the initial step of the walker differently. The model was partly motivated by the properties of financial time series and tested on empirical data extracted from the Warsaw stock exchange since it offers an opportunity to study in an unbiased way several features of the stock exchange in its early stages. We extended the continuous-time random walk formalism but the (generalized) waiting-time distribution (WTD) and sojourn probability density still play a fundamental role. We considered a one-dimensional, non-Brownian random walk where the walker moves, in general, with a velocity that assumes a different constant value between the successive turning points, i.e. the velocity is a piecewise constant function. So far the models which have been developed take only one chosen value of this velocity into account and therefore are unable to consider more realistic stochastic time series. Moreover, our model is a kind of Levy walk where we assume a hierarchical, self-similar in the stochastic sense, spatio-temporal representation of WTD and sojourn probability density. The Weierstrass–Mandelbrot walk model makes it possible to analyse both the structure of the Hurst exponent and the power-law behaviour of kur-

tosis. This structure results from the hierarchical, spatio-temporal coupling between the walker displacement and the corresponding time of the walks. The analysis makes use of both the fractional diffusion and the super-Burnett coefficients. We constructed the diffusion phase diagram which distinguishes regions occupied by classes of different universality. We study only such classes which are characteristic of stationary situations. We proved that even after taking a moving averaging of the stochastic time series which makes results stationary in the sense that they are independent of the beginning moment of the random walk, it is still possible to see the non-Gaussian features of the basic stochastic process. We thus have a model ready for describing data presented, e.g., in the form of moving averages. This operation is often used for stochastic time series, especially financial ones. Based on the hierarchical representation of WTD we introduce an efficient Monte Carlo algorithm which makes a numerical simulation of individual runs of stochastic time series possible; this facilitates the study of empirical stochastic time series.

Quantitative Finance 3 (2003), 201.

Dynamic structural and topological phase transitions on the Warsaw Stock Exchange: A phenomenological approach

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We study the crash dynamics of the Warsaw Stock Exchange (WSE) by using the Minimal Spanning Tree (MST) networks. We find the transition of the complex network during its evolution from a (hierarchical) power law MST network, representing the stable state of WSE before the recent worldwide financial crash, to a superstar-like (or superhub) MST network of the market decorated by a hierarchy of trees (being, perhaps, an unstable, intermediate market state). Subsequently, we observed a transition from this complex tree to the topology of the (hierarchical) power law MST network decorated by several star-like trees or hubs. This structure and topology represent, perhaps, the WSE after the worldwide financial crash, and could be considered to be an aftershock. Our results can serve as an empirical foundation for a future theory of dynamic structural and topological phase transitions on financial markets.

arXiv:1301.6506 [q-fin.ST]

Excess Noise for Driven Diffusive Systems

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We investigate the steady-state scattering function for driven diffusive systems with a single conserved density. In one dimension, density fluctuations spread as $t^{2/3}$, i.e., faster than the diffusive $t^{1/2}$, for large time t . The corresponding excess noise in the current-current correlation diverges as $\omega^{-1/3}$ for small frequency ω . Monte Carlo simulation results for a driven hard-core lattice gas confirm these results. $d=2$ is the borderline dimension with marginally nondiffusive behavior; for $d>2$, the spread is diffusive with anisotropic long-time-tail corrections.

Physical Review Letters 54, 2026 (1985).

Article

Chemical diffusion in the lattice gas of non-interacting particles

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The occupancy correlation function of non-interacting particles hopping on a lattice, with exclusion of double occupancy, is rederived in a concise way. The resulting chemical diffusion coefficient is discussed.

Physics Letters A 81, 239 (1981).

Article

Study of the non-linear autocorrelations within the Gaussian regime

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In this work we extend the recently considered toy model of Weierstrass or Lévy walks with varying velocity of the walker [1] by introducing a more realistic possibility that the walk can be occasionally intermitted by its momentary localization; the localizations themselves are again described by the Weierstrass or Lévy process. The direct empirical motivation for developing this combined model is, for example, the dynamics of financial high-frequency time series or hydrological and even meteorological ones where variations of the index are randomly intermitted by flat intervals of different length exhibiting no changes in the activity of the system. This combined Weierstrass walks was developed in the framework of the non-separable generalized continuous-time random walk (GCTRW) formalism developed recently [2]. This approach makes it possible to study by stochastic simulations the whole spatial-temporal range while analytically we can study only the initial, pre-asymptotic and asymptotic regions (but not the intermediate one). Our approach is possible since the Weierstrass walks is a geometric superposition of regular random walks each of which can be simply treated by stochastic simulations. This non-Markovian two-state (walking-localization) model makes possible to cover by the unified treatment a broad band of known up to now types of non-biased dif-

fusion from the dispersive one over the normal, enhanced, ballistic, and hyperdiffusion up to the Richardson law of diffusion which defines here a part of the borderline which separates the latter from the ‘Lévy ocean’ where the total mean-square displacement of the walker diverges. We observed that anomalous diffusion is characterized here by three fractional exponents: temporal one characterizing the localized state and two, temporal and spatial ones, characterizing the walking state. By considering successive dynamic (even) exponents we constructed a series of different diffusion phase diagrams on the plane defined by the spatial and temporal fractional dimensions of the walking state. To adapt the model to the description of empirical data (or discrete time series) which are collected with a discrete time-step we used in the continuous-time series produced by the model a discretization procedure. We observed that such a procedure generates, in general, long-range non-linear autocorrelations even in the Gaussian regime, which appear to be similar to those observed, e.g., in the financial time series [3–6], although single steps of the walker within continuous time are, by definition, uncorrelated. This suggests a surprising explanation alternative to the one proposed very recently (cf. [7] and Refs. therein) although both approaches involve related variants of the well-known CTRW formalism applied yet in many different branches of knowledge [8–10].

The European Physical Journal B-Condensed Matter and Complex Systems 33 (2003), 495.

Article

Anomalous transport and diffusion versus extreme value theory

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In the present work we match the biased hierarchical continuous-time random flight (HCTRF) on a regular lattice (based on hierarchical waiting-time distribution) and the extreme event theory (EVT). This approach extends the understanding of the anomalous transport and diffusion (for example, found in some amorphous, vitreous solids as well as in conducting and light-emitting organic polymers). Both independent approaches were developed in terms of random-trap or valley model where the disorder of energy landscape is exponentially distributed while the corresponding mean residence times in traps obey the power-law. This type of disorder characterizes several amorphous (even used commercially) materials which makes it possible to apply the HCTRF formalism. By using the EVT we additionally show that the rare (stochastic) events are indeed responsible for the transport and diffusion in these materials.

Physica A 357 (2005), 282.

Diffusion in concentrated lattice gases. II. Particles with attractive nearest-neighbor interaction on three-dimensional lattices

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The diffusion process of particles in concentrated lattice gases is investigated, with the assumption of a fcc lattice and nearest-neighbor attraction (but double occupancy of sites being forbidden). Both the self-diffusion of tagged particles and the collective diffusion, by which concentration fluctuations decay, are studied. Apart from a mean-field treatment, the various diffusion coefficients are estimated by Monte Carlo techniques and interpreted in terms of static long- and short-range order (i.e., unmixing) occurring in this model system. Collective diffusion is studied by direct simulation of linear response to wave-vector-dependent "fields." Near the critical temperature, pronounced critical slowing down of the collective diffusion coefficient is observed. The self-diffusion constant stays finite but exhibits a singularity of its slope. The correlation factor for self-diffusion is found to be practically independent of temperature. A qualitative discussion of the behavior inside of the mixed-phase region is also given.

Physical Review B 26, 2967 (1982).

Mean square displacement of a tracer particle in a hard-core lattice gas

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A simple theory is developed for hopping diffusion in hard-core lattice gases. Its predictions for the mean square displacement of a tracer particle as a function of time are found to be in good agreement with the results of Monte Carlo simulations on a quadratic lattice. For long times a logarithmic term is found in accord with mode-coupling predictions. Our predictions for the tracer diffusion coefficient agree with those of Tahir-Kheli.

Physical Review Letters 55, 238 (1985).

Diffusion in concentrated lattice gases: Intermediate incoherent dynamical scattering function for tagged particles on a square lattice

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The time-dependent position self-correlation function of the particles of two-dimensional lattice gases was estimated by Monte Carlo simulations at various concentrations c . Two exponential decay modes were identified for $c < 0.5$ and three modes for $c > 0.5$ in the resulting intermediate incoherent dynamical scattering function. Published microscopic theories for that quantity are found to be at variance with the numerical results at larger wave vectors. A phenomenological description of the results is achieved by two-state models for $c < 0.5$ and three-state models for $c > 0.5$. A detailed physical interpretation of the model parameters, however, is an open question.

Physical Review B 41 (1990), 2784.

Structural and topological phase transitions on the German Stock Exchange

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We find numerical and empirical evidence for dynamical, structural and topological phase transitions on the (German) Frankfurt Stock Exchange (FSE) in the temporal vicinity of the worldwide financial crash. Using the Minimal Spanning Tree (MST) technique, a particularly useful canonical tool of the graph theory, two transitions of the topology of a complex network representing the FSE were found. The first transition is from a hierarchical scale-free MST representing the stock market before the recent worldwide financial crash, to a superstar-like MST decorated by a scale-free hierarchy of trees representing the market's state for the period containing the crash. Subsequently, a transition is observed from this transient, (meta)stable state of the crash to a hierarchical scale-free MST decorated by several star-like trees after the worldwide financial crash. The phase transitions observed are analogous to the ones we obtained earlier for the Warsaw Stock Exchange and more pronounced than those found by Onnela-Chakraborti-Kaski-Kertész for the S&P 500 index in the vicinity of Black Monday (October 19, 1987) and also in the vicinity of January 1, 1998. Our results provide an empirical foundation for the future theory of dynamical, structural and topological phase transitions on financial markets.

Fractional Market Model and its Verification on the Warsaw Stock Exchange

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We analyzed the rising and relaxation of the *cusp*-like local peaks superposed with oscillations which were well defined by the Warsaw Stock Exchange index WIG in a daily time horizon. We found that the falling paths of all index peaks were described by a generalized exponential function or the Mittag-Leffler (ML) one superposed with various types of oscillations. However, the rising paths (except the first one of WIG which rises exponentially and the most important last one which rises again according to the ML function) can be better described by bullish anti-bubbles or inverted bubbles.²⁻⁴ The ML function superposed with oscillations is a solution of the nonhomogeneous fractional relaxation equation which defines here our Fractional Market Model (FMM) of index dynamics which can be also called the Rheological Model of Market. This solution is a generalized analog of an exactly solvable fractional version of the Standard or Zener Solid Model of viscoelastic materials commonly used in modern rheology.⁵ For example, we found that the falling paths of the index can be considered to be a system in the intermediate state lying between two complex ones, defined by short and long-time limits of the Mittag-Leffler function; these limits are given by the Kohlrausch-Williams-Watts (KWW) law for the initial times, and the power-law or the Nutting law for asymptotic time. Some rising paths (i.e., the bullish anti-bubbles) are a kind of log-periodic oscillations of the market in the bullish state initiated by a crash. The peaks of the index can be viewed as precritical or pre-crash ones since:

- (i) the financial market changes its state too early from the bullish to bearish one before it reaches a scaling region (defined by the diverging power-law of return per unit time), and
- (ii) they are affected by a finite size effect.

These features could be a reminiscence of a significant risk aversion of the investors and their finite number, respectively. However, this means that the scaling region (where the relaxations of indexes are described by the KWW law or stretched exponential decay) was not observed. Hence, neither was the power-law of the instantaneous returns per unit time observed. Nevertheless, criticality or crash is in a natural way contained in our FMM and we found its "finger print".

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<http://www.worldscientific.com/doi/abs/10.1142/S012918310801225>
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International Journal of Modern Physics C 19 (2008), 453.

Bose-Einstein condensation shown by Monte Carlo simulation

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A Monte Carlo algorithm was constructed for calculating statistical physics quantities characterizing an ideal bosonic lattice gas. To demonstrate the possibilities of the algorithm the Bose-Einstein condensation was simulated within a three-dimensional isotropic harmonic oscillator where, in the frame of a canonical ensemble, average occupancy of the ground state and specific heat was calculated versus temperature and number of bosons. The algorithm can be further applied for studying both the static properties of ideal bosons within other trapping potentials and the relaxation of the system to the condensate.

Computer Physics Communications 121-122 (1999), 586.

Remarks on the possible universal mechanism of the non-linear long-term autocorrelations in financial time-series

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Random walk on a linear chain with a quenched distribution of jump lengths

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We study the random walk of a particle on a linear chain, where a jump length 1 or 2 is assigned randomly to each lattice site with probability p_1 and $p_2=1-p_1$, respectively. We find that the probability peff_1 for the particle to be at a site with jump length 1 is different from p_1 , which causes the diffusion coefficient D to differ from the mean-field result. A theory is developed that allows us to calculate peff_1 and D for all values of p_1 . In the limit $p_1 \rightarrow 0$, the theory yields a nonanalytic dependence of peff_1 on p_1 , $\text{peff}_1 \propto -p_2 \ln p_1$

Physical Review E 55 (1997), 71.

Modified Fermi-Dirac Statistics of Fermionic Lattice Gas by the Back-Jump Correlations

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The influence of the Bardeen-Herring back-jump correlations on the Fermi-Dirac statistics of the one-dimensional nonhomogeneous fermionic lattice gas is studied by the Monte Carlo simulation technique and semianalytically. The resulting distribution is obtained, exhibiting increased population of the lower levels in comparison to the Fermi-Dirac statistics.

Journal of Statistical Physics 62 (1991), 389.

Thermal neutron scattering from the hydrogen-metal systems in terms of general multi-sublattice jump diffusion model - II: Remarks on hydrogen diffusion in the α -phase of Nb-H

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The two-parameter Multi-Sublattice Jump Diffusion Model (MSJD), presented in Part I [1], is applied to Hydrogen diffusion in the α -phase of Nb-H. It is proved, mainly on the basis of quasielastic neutron scattering experiments on a single crystal of NbH 0.09 [2] at $T = 510$ K, that diffusion occurs as a result of jumps between adjacent and second nearest neighbor tetrahedral interstitial sites.

Journal of Physics and Chemistry of Solids 38 (1977), 747

Simple molecular mechanisms of heat transfer: Debye relaxation versus power-law

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We study a simple molecular model (at a coarse-grain level) as a basis of irreversible heat transfer through a diathermic partition. The partition separates into two adjacent parts a box containing ideal point particles that communicate only through this partition. We provide the basic mechanism of energy transfer between the left- and right-hand side gas samples by assuming equipartition of kinetic energy of all outgoing particles colliding with the partition at a given time. We analyse and compare two essentially different cases: (A) the reference one, where we assume that the border walls of the box and the diathermic partition can randomize the direction of motion

of rebounding particles, and (B) the case where we assume the mirror collisions of particles with the border walls and the partition. In both cases the rebounding of the particles from border walls is elastic. The above introduced assumptions allow us to numerically simulate and analytically consider, for example, the relaxation of temperatures of both gas samples and the entropy of the system. However, in both cases the long-time relaxation is essentially different since in case (A) it is an exponential one, while in case (B) it seems to be a power-law relaxation. The obtained results well agree in case (A) with the predictions of the phenomenological, linear theory of irreversible thermodynamic processes. However, to describe case (B) a version of this theory had to be developed which assumes time-dependence of heat conductivity; it describes the relaxation of the system far from equilibrium. The explanation of the results obtained in this case is, nevertheless, an intriguing problem.

Physica A 352 (2005), 347.

Stock market context of the Lévy walks with varying velocity

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We developed the most general Lévy walks with varying velocity, shorter called the Weierstrass walks (WW) model, by which one can describe both stationary and non-stationary stochastic time series. We considered a non-Brownian random walk where the walker moves, in general, with a velocity that assumes a different constant value between the successive turning points, i.e., the velocity is a piecewise constant function. This model is a kind of Lévy walks where we assume a hierarchical, self-similar in a stochastic sense, spatio-temporal representation of the main quantities such as waiting-time distribution and sojourn probability density (which are principal quantities in the continuous-time random walk formalism). The WW model makes possible to analyze both the structure of the Hurst exponent and the power-law behavior of kurtosis. This structure results from the hierarchical, spatio-temporal coupling between the walker displacement and the corresponding time of the walks. The analysis uses both the fractional diffusion and the super Burnett coefficients. We constructed the diffusion phase diagram which distinguishes regions occupied by classes of different universality. We study only such classes which are characteristic for stationary situations. We thus have a model ready for describing the data presented, e.g., in the form of moving averages; the operation is often used for stochastic time series, especially financial ones. The model was inspired by properties of financial time series and tested for empirical data extracted from the Warsaw stock exchange since it offers an opportunity to study in an unbiased way several features of stock exchange in its early stage.

Physica A 314 (2002), 786.

Tracer diffusion on two coupled lines: The long-time tail of the velocity autocorrelation function compared to the mode-coupling prediction

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Mode-coupling predictions for the long-time behavior of the mean-square displacement of tracer particle diffusion in a quasi-one-dimensional lattice-gas model are compared with the results of Monte Carlo simulations. Furthermore, the internal consistency of an approximate theory is investigated under the assumption that mode-coupling theory is valid.

Physical Review B 32 (1985), 466.

Distribution for Fermionic Discrete Lattice Gas within the Canonical Ensemble

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The distinct deviations from the Fermi-Dirac statistics ascertained recently at low temperatures for a one-dimensional, spinless fermionic discrete lattice gas with conserved number of noninteracting particles hopping on the non-degenerated, well-separated single-particle energy levels are studied in numerical and theoretical terms. The generalized distribution is derived in the form $n(h) = \{Y(h) \exp[(e(h) - \mu)/kT] + 1\}^{-1}$ valid even in the thermodynamic limit, when the discreteness of the energy levels is kept. This distribution demonstrates good agreement with the data obtained numerically both by the canonical partition function technique and by Monte Carlo simulation.

Journal of Statistical Physics 65 (1991), 813.

Monte Carlo Simulations of Lattice Gases Exhibiting Quantum Statistical Distributions

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A non-interacting lattice gas with order preservation in a constant external field is studied by numerical and analytical methods. The equilibrium distribution is of the Bose-Einstein type. If additional hard-core repulsion is imposed, it becomes a distribution of Fermi-Dirac type. When relaxing the order preservation condition the classical Boltzmann distribution is recovered.

International Journal of Modern Physics C 02 (1991), 450.

Modeling of super-extreme events: An application to the hierarchical Weierstrass-Mandelbrot Continuous-time Random Walk

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We analytically demonstrate and numerically simulate two utmost cases of dragon-kings' impact on the (unnormalized) velocity autocorrelation function (VACF) of a complex time series generated by stochastic random walker. The first type of dragon-kings corresponds to a sustained drift whose duration time is much longer than that of any other event. The second type of dragon-kings takes the form of an abrupt shock whose amplitude velocity is much larger than those corresponding to any other event. The stochastic process in which the dragon-kings occur corresponds to an enhanced diffusion generated within the hierarchical Weierstrass-Mandelbrot Continuous-time Random Walk (WM-CTRW) formalism. Our analytical formulae enable a detailed study of the impact of the two super-extreme events on the VACF calculated for a given random walk realization on the form of upward deviations from the background power law decay present in the absence of dragon-kings. This allows us to provide a unambiguous distinction between the super-extreme dragon-kings and 'normal' extreme "black swans". The results illustrate diagnostic that could be useful for the analysis of extreme and super-extreme events in real empirical time series.

The European Physical Journal Special Topics 205 (2012) 27-52.

Biased random walk on a biased random walk

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We consider the random walk of a particle along topologically linear channels under the influence of a uniform drift force. The channels are generated by the usual biased random walk procedure. The resulting mean and mean-square displacements of a particle are discussed.

Physica A 171 (1991), 43.

Possible origin of the non-linear long-term autocorrelations within the Gaussian regime

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In this work we extend the recently considered toy model of Weierstrass or Lévy walks with varying velocity of the walker (Quantitative Finance 3 (2003) 201; Chem. Phys. 284 (2002) 481; Comp. Phys. Comm. 147 (2002) 565; Phys. A 264 (1999) 84; Phys. A 264 (1999) 107) by introducing a more realistic possibility that the walk can be occasionally intermitted by its momentary localization; the localizations themselves are again described by the Weierstrass or Lévy process. The direct empirical motivation for developing this combined model is, for example, the dynamics of financial high-frequency time series or hydrological and even meteorological ones where variations of the index are randomly intermitted by flat intervals of different length exhibiting no changes in the activity of the system. This combined Weierstrass walks was developed in the framework of the non-separable generalized continuous-time random walk formalism developed very recently (Lecture Notes Comput. Sci. 2657 (2003) 407; Eur. Phys. J. B 33 (2003) 495). This non-Markovian two-state (walking-localization) model makes possible to cover by the unified treatment a broad band of known up to now types of non-biased diffusion from the dispersive one over the normal, enhanced, ballistic, and hyperdiffusion up to the Richardson law of diffusion which defines here a part of the borderline which separates the latter from the ‘Lévy ocean’ where the total mean-square displacement of the walker diverges. We observed that anomalous diffusion is characterized here by three fractional exponents: one (temporal) characterizing the localized state and two (temporal and spatial) characterizing the walking one. By considering successive dynamic (even) exponents we constructed a series of different diffusion phase diagrams on the plane defined by the temporal and spatial (partial) fractional (dynamic) exponents characterizing the walking state. To adapt the model to the description of empirical data (the discrete time series), which are collected with a discrete time step, we used in the continuous-time series produced by the model a discretization procedure. We observed that such a procedure generates, in general, long-range non-linear autocorrelations even in the Gaussian regime, which appear to be similar to those observed, e.g., in the financial time series (Phys. A 287 (2000) 396; Phys. A 299 (2001) 1; Phys. A 299 (2001) 16; Phys. A 299 (2001) 16), although single steps of the walker within continuous time are, by definition, uncorrelated. This suggests a surprising origin of long-range non-linear autocorrelations alternative to the one proposed very recently (cf. Mosaliver et al. (Phys. Rev. E 67 (2003) 021112) and refs. therein) although both approaches involve related variants of the well-known continuous-time random walk formalism applied yet in many different branches of knowledge (Phys. Rep. 158 (1987) 263; Phys. Rep. 195 (1990) 127; in: A. Bunde, S. Havlin (Eds.), Fractals in Science, Springer, Berlin, 1995, p. 1).

Physica A 330 (2003), 177.

Applications of statistical mechanics to non-brownian random motion

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We analysed discrete and continuous Weierstrass–Mandelbrot representations of the Lévy flights occasionally interrupted by spatial localizations. We chose the *discrete* representation to easily detect by Monte Carlo simulation which stochastic quantity could be a candidate for describing the real processes. We found that the particle propagator is able to reveal surprisingly close, stable long-range algebraic tail. Unfortunately, long flights present in the system make, in practice, the particle mean-square displacement an irregular step-like function; such a behavior was expected since it is an experimental reminiscence of divergence of the mean-square displacement, predicted by the theory. We developed the *continuous* representation in the context of random motion of a particle in an amorphous environment; we established a correspondence between the stochastic quantities of both representations in which the latter quantities contain some material constants. The material constants appear due to the thermal average of the space-dependent stretch exponent which defines the probability of the particle passing a given distance. This averaging was performed for intermediate or even high temperatures, as well as for low or even intermediate internal friction regimes where long but *not* extremely long flights are readily able to construct a significant part of the Lévy distribution. This supplies a kind of self-cut-off of the length of flights. By way of *example*, we considered a possibility of observing the Lévy flights of hydrogen in amorphous low-concentration, high-temperature Pd₈₅Si₁₅H_{7.5} phase; this conclusion is based on the results of a real experiment (Driesen et al., in: Janot et al. (Eds.), Atomic Transport and Defects in Metals by Neutron Scattering, Proceedings in Physics, Vol. 10, Springer, Berlin, 1986, p. 126; Richter et al., Phys. Rev. Lett. 57 (1986) 731; Driesen, Doctoral Thesis, Antwerpen University, 1987), performed by detecting the incoherent quasielastic scattering of thermal neutrons. We emphasize that the observed HWHM $\propto k^\beta$, where exponent β is distinctly smaller than 2, could be caused by these long flights of hydrogen.

Physica A 274 (1999), 67.

Real-time numerical simulation of the Carnot cycle

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We developed a highly interactive, multi-windows Java applet which made it possible to simulate and visualize within any platform

and internet the Carnot cycle (or engine) in a real-time computer experiment. We extended our previous model and algorithm (Galant *et al* 2003 *Heat Transfer, Newton's Law of Cooling and the Law of Entropy Increase Simulated by the Real-Time Computer Experiments in Java (Lecture Notes in Computer Science vol 2657)* pp 45–53, Gall and Kutner 2005 Molecular mechanisms of heat transfer: Debye relaxation versus power-law *Physica A* **352** 347–78) to simulate not only the heat flow but also the macroscopic movement of the piston. Since in reality it is impossible to construct a reversible Carnot engine, the question arises whether it is possible to simulate it at least in a numerical experiment? The positive answer to this question which we found is related to our model and algorithm which make it possible to omit the many-body problem arising when many gas particles simultaneously interact with the mobile piston. As usual, the considerations of phenomenological thermodynamics began with a study of the basic properties of heat engines, hence our approach, besides intrinsic physical significance, is also important from the educational, technological and even environmental points of view.

European Journal of Physics 26 (2005), 673.

Article

Quantum statistics and discreteness. Differences between the canonical and grand canonical ensembles for a fermionic lattice gas

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Studying the electronic properties of small metallic particles Kubo and subsequently Denton, Mühlischlegel and Scalapino recognized the surprising effect that the mean occupation numbers of noninteracting fermions (electrons) are not given within the canonical ensemble by the usual expression of Fermi-Dirac statistics. The fermions behave as if they are colder and observed deviations are most significant in the vicinity of the Fermi level. The results of our Monte Carlo simulations, as well as our complementary numerical and analytical results are in very good agreement with the predictions of Denton *et al*. We found that the effect vanishes for very small and very large spacing between the energy levels in comparison to the thermal energy $k_B T$. Associated with this effect are the particle-hole correlations. Extending the method used by Denton *et al*. we derive an analytical formula for the particle-hole correlation functions whose predictions are again in very good agreement with our numerical results. The low-temperature aspects of the effect are considered by applying a two-state model. The role of the finite-size effect is also discussed.

Annalen der Physik 507 (1995), 646.

Article

Tracer diffusion in honey-comb lattice correlations over several consecutive jumps

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The hopping of a tracer in a two-dimensional lattice gas model with low coordination number is examined. The model consists of noninteracting particles except that double sites occupancy is forbidden. An arbitrary concentration, c , on a honey-comb lattice is assumed. The correlation factor, $f(c)$, is estimated by the Monte Carlo simulation, and compared with theoretical expressions. The correlations over two successive jumps of a tracer are in general insufficient to account for $f(c)$ data. Inclusion of correlations over three and four jumps improves the description. The theory of Nakazato and Kitahara [1] gives a satisfactory description of the data.

Solid State Ionics 9-10 (1983), 1409.

Article

Spatio-temporal coupling in the continuous-time Lévy flights

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Continuous-time Lévy flights (CTLF) are reconsidered in the framework of non-separable continuous-time random walk (CTRW) model in order to properly treat the spatio-temporal relations in terms of the self-similar structure of the Lévy process. Accordingly, a novel spatio-temporal coupling is introduced by assuming that in each order of the structure the probability density for the flight and that for waiting are joined. As a result of competition between flights and waitings an enhanced, dispersive or normal diffusion was recognized in distinction from the prediction of the commonly used separable CTRW model where, e.g., the enhanced diffusion is lost and dispersive one is strongly limited. This renewed CTFL approach offers a possibility to properly model the time-dependence for any fractional (critical) wandering of jump type, for example, it seems that the hydrogen diffusion in some metallic glasses or nanocompounds has chance to be described by our approach.

Solid State Ionics 119 (1999), 323.

Article

Determination of the chemical diffusion coefficient by Monte Carlo simulation of the center-of-mass propagation

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The extended Zwergermethod of determining the chemical diffusion coefficient, for lattice gas conventional diffusion in equilibrium and superdiffusion in stationary state, is verified by Monte Carlo simulation of the centre-of-mass propagation. It has been found that the chemical diffusion coefficient for superdiffusion in one-dimensional, noninteracting lattice gas with the exclusion of double occupancy is concentration independent like in conventional diffusion. Moreover, it has been proved that (chemical) superdiffusion of uncorrelated particles is exactly described in this model by the Burgers nonlinear diffusion equation.

Zeitschrift für Physik B 86 (1992), 461.

Article

Tracer Diffusion in Concentrated Lattice Gas Models. Rectangular Lattices with Anisotropic Jump Rates

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An approximate theory is developed for tracer diffusion in rectangular lattice gas models with anisotropic jump rates to neighboring unoccupied sites in different directions. Comparison with Monte Carlo simulations on quadratic lattices with several ratios for the jump rates in orthogonal directions shows a satisfactory agreement in all cases investigated.

Journal of Statistical Physics 49 (1987), 1043.

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